Before Algebra Is Named: Heidegger, Anonymous Operations, Mathematical Disclosure, and the Limits of Abiogenic Algebra

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Algebraic discovery is often described as the extraction of laws from initially anonymous operations, but anonymity does not imply absence of meaning. A finite operation table may withhold familiar names such as addition, subtraction, conjunction, or composition while still presenting a fully defined rule embedded in practices of lookup, substitution, comparison, and repetition. This paper examines the philosophical and mathematical consequences of that distinction. Drawing on model theory, universal algebra, structuralism, and Heidegger's account of practical involvement, it argues that an operation is never encountered as wholly uninterpreted data. Before it becomes an explicit mathematical object, it already functions within a structured activity and a human project of inquiry. Algebraic Abiogenesis therefore cannot literally describe the birth of algebra from non-algebra. Its defensible subject is the transition from implicit operational intelligibility to explicit equational articulation. The paper distinguishes operational definition, historical naming, structural recognition, and mathematical discovery; analyzes how observers, computational substrates, and inherited practices condition what can appear as algebra; and proposes mathematical disclosure as a more precise framework for understanding operation-first theory formation, artificial mathematical discovery, and the emergence of formal objects from practiced relations.

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1. The Problem of Anonymous Mathematical Operations

The phrase “anonymous operation” appears to promise a beginning before interpretation. A symbol such as $\diamond$ is introduced, a table specifies how it acts, and familiar names such as addition, subtraction, conjunction, multiplication, or composition are withheld. The investigator generates expressions, compares outputs, and attempts to discover the operation’s laws without prejudging its mathematical identity. At first glance, this seems to provide a neutral starting point from which algebraic structure can emerge.

The neutrality is only partial. The operation may be anonymous because its conventional name and classification are unknown, but it is already mathematically specified. It has an arity, a domain, a codomain, a determinate output for every permitted input, and a role within procedures of substitution, iteration, and comparison. Its symbol may be unfamiliar, yet its use is governed from the beginning.

This distinction matters because claims about mathematical discovery can become inflated when anonymity is confused with absence of definition. If $\diamond$ is presented by a complete table, the operation does not come into existence when its laws are later recognized. It was present from the first lookup. What emerges is not the operation itself, but an explicit account of its regularities, classifications, derived structures, and relations to established mathematics.

The central problem is therefore to determine what kind of ignorance anonymity represents. The investigator may not know the operation’s historical name, shortest axiom basis, standard representation, or relationship to a familiar algebraic theory. These are genuine forms of mathematical ignorance. They do not imply that the operation is undefined, structureless, or devoid of practical meaning.

Operation-first discovery remains valuable, but its claims must be stated precisely. It can reveal laws before those laws are deliberately selected as targets. It can recover structure under unfamiliar notation. It can expose how much algebraic organization follows from a finite operational specification. It cannot begin from a complete absence of mathematical presupposition.

1.1 Anonymous Does Not Mean Undefined

Suppose a finite carrier is given:

$A = \{a, b, c\}$,

together with a symbol $\diamond$ and a table assigning one element of A to every ordered pair in $A \times A$. The symbol $\diamond$ may have no familiar interpretation, but the operation is fully defined extensionally:

$\diamond : A \times A \rightarrow A$.

For every x and y in A, the value $x \diamond y$ is determined. Nothing remains undefined at the level of individual applications.

Anonymity concerns recognition, not existence. The investigator may not know whether $\diamond$ is isomorphic to addition modulo three, subtraction, a quasigroup operation, an opposite multiplication, or a derived operation from some better-known algebra. Yet the table still determines exactly what $\diamond$ does.

This resembles the difference between encountering an unfamiliar machine and encountering no machine at all. A device may have an unknown purpose while possessing a definite mechanism. One can operate it, observe its behavior, and infer its function. Ignorance of purpose does not imply indeterminacy of action.

Likewise, an anonymous mathematical operation is better described as a fully specified operation under withheld interpretation. Its historical identity, conventional notation, and theoretical classification remain open. Its extensional behavior does not.

This distinction prevents a common conceptual error. Suppose the investigation establishes

$(x \diamond y) \diamond z = x \diamond (y \diamond z)$.

Associativity has not been created by the act of recognition. The already defined operation has been shown to satisfy an associative law. The law may have been unknown to the investigator, but it was true of the table before the investigation began.

What changed was the investigator's knowledge, not the operation's behavior.

1.2 The Operation Table as a Complete Extensional Definition

An operation table defines by listing. Each ordered pair of inputs is assigned an output. For a finite carrier, this gives a complete extensional specification.

Such a definition differs from an intensional definition. An intensional definition gives a formula, construction, rule, or conceptual characterization. For example, the formula

$x \diamond y = y - x \bmod 3$

may explain the same operation represented by a three-element table.

The table and the formula can determine identical input-output behavior while presenting the operation differently. The table states every value separately. The formula compresses those values into a recognizable rule.

The table therefore tells us everything the operation does without necessarily telling us why its values form that pattern. It is complete behaviorally but may remain conceptually opaque.

This gap is precisely where mathematical discovery can occur. The investigator seeks a shorter, more general, or more illuminating account of the table. A repeated diagonal value may reveal a definable constant. A recurring term collision may reveal an identity. A small collection of identities may explain a large number of individual evaluations.

The movement is therefore:

complete table → recognized regularity → compressed law → structural classification.

The table is not meaningless data. It is already organized as a total function. But it may contain more information than can be readily understood from inspection. Extracting a compact law system from that exhaustive specification is a genuine act of mathematical compression.

This clarification preserves the legitimacy of operation-first discovery. The discovery is not the creation of the operation. It is the transition from an exhaustive presentation to a structural account.

1.3 Withheld Interpretation and Delayed Classification

Withholding interpretation is a methodological discipline. It prevents the investigator from testing only those properties expected of a familiar operation.

If a table is introduced as addition, attention will naturally turn toward associativity, commutativity, identity, and inverses. If it is introduced as lattice meet, the investigator will expect idempotence, commutativity, associativity, absorption, and order-theoretic behavior. The name directs the search before any calculation begins.

Using $\diamond$ delays that direction. Expressions can be generated mechanically and their outputs compared without privileging a known theory. This may reveal laws that do not fit the expected profile of any familiar operation.

The method separates two stages:

law detection → historical classification.

At the first stage, one records that two terms agree under every assignment. At the second, one asks whether the resulting equality is known as associativity, mediality, absorption, cancellation, distributivity, or a consequence of another established law.

This separation is epistemically useful. It reduces confirmation bias and allows an operation to display an unexpected combination of properties before it is placed inside a familiar conceptual category.

The delay does not eliminate prior knowledge. The investigator still understands binary operations, variables, term formation, substitution, equality, and finite enumeration. One layer of interpretation has been withheld while several deeper layers remain active.

Delayed classification also prevents premature closure. A table may satisfy some group-like identities while also exhibiting behavior not naturally emphasized by group theory. Naming the operation too quickly may cause its less familiar features to be overlooked.

Nevertheless, delayed classification cannot become permanent avoidance of comparison. Once identities have been extracted, the operation must be tested against known mathematics. Otherwise, a rediscovery presented in unfamiliar notation may be mistaken for novelty.

The proper discipline is therefore:

first withhold the name, then earn the classification.

1.4 Why Unfamiliar Notation Creates an Illusion of Conceptual Neutrality

Symbols influence thought. A plus sign suggests addition. A centered dot suggests multiplication. A wedge suggests conjunction or meet. A small circle suggests composition. Replacing these symbols with ⋄ removes many familiar associations.

But removing familiar associations does not create complete conceptual neutrality. The new symbol still occupies a defined grammatical role. It appears between two expressions, accepts ordered inputs, permits nesting, produces an output, and may be evaluated repeatedly.

Thus the notation is neutral only relative to conventional naming. It is not neutral relative to algebraic form.

The expression

$x \diamond y$

already presents ⋄ as a binary combination. The expression

$(x \diamond y) \diamond z$

presupposes that outputs may become new inputs. Variables presuppose substitution. Equations presuppose a criterion of sameness. The table presupposes distinct input positions and a stable lookup procedure.

Even the choice of the diamond symbol is not wholly neutral. Mathematically trained readers recognize it as a conventional placeholder for an operation whose familiar meaning has been intentionally suppressed.

The illusion of neutrality arises because the absence of a familiar name is conspicuous while the deeper formal scaffolding remains unnoticed. The observer sees that the symbol is not + or × and may overlook that the entire setting is already algebraic.

This is not a reason to abandon neutral notation. Neutral notation remains valuable for controlling expectations. It is a reason to describe its role accurately.

The symbol ⋄ removes one interpretive layer. It does not remove interpretation itself.

1.5 The Limits of Operation-First Language

Operation-first language can postpone classification, but it cannot escape the structure required to make an operation intelligible.

To present ⋄ at all, one must specify or assume:

- a collection of distinguishable elements;
- an arity;
- an ordering of argument positions;
- a domain and codomain;
- a deterministic rule or table;
- equality of elements;
- repeatable evaluation;
- permissible composition;
- variable substitution.

These are already mathematical commitments.

The chosen language also limits what can be discovered. A system containing one total binary operation is naturally suited to finding equational properties of binary algebras. It is not automatically equipped to detect partiality, probability, continuity, topology, order, geometry, infinitary behavior, or observational uncertainty.

A property that cannot be expressed in the available grammar cannot appear as a term identity. A constant may remain invisible unless constants are allowed or can be defined from variable terms. A unary operation may remain hidden unless it can be produced by fixing an argument or composing available operations. A relation may never become visible if the observational system records only outputs.

Operation-first discovery must therefore be understood as inquiry within a specified field of possible disclosure. It asks what becomes explicit when one particular kind of behavior is examined using one particular language of observation.

Its strongest legitimate description is:

fully specified but unclassified operation → explicit laws, derived structure, and mathematical recognition.

This is not a transition from meaninglessness to mathematics. It is a transition from practical and extensional specification to theoretical articulation.

The operation exists before it is classified. Its regularities hold before they are named. The investigator's work is to make those regularities explicit, compress them into laws, compare them with known structures, and determine whether anything mathematically new remains after recognition.

2. From Marks on Paper to Mathematical Objects

A pencil-and-paper experiment can appear philosophically simpler than a computational one. There is no processor instruction set, programming language, operating system, or hidden software library. The investigator draws several marks, writes an operation table, and performs substitutions by hand. It may therefore seem that the mathematical structure has been reduced to its most elementary form.

The reduction is real but incomplete. Pencil and paper remove many layers of technological mediation while preserving the deeper practices that make mathematics possible. The marks must be distinguishable. Their locations must remain stable. A table must be read according to a convention. The investigator must remember which input occupies the row and which occupies the column. Repeated marks must be treated as instances of the same symbol. Outputs must be reusable as inputs. Results must be compared across cases.

These conditions are not later discoveries. They are already active when the first mark is written. The experiment therefore does not begin with meaningless material and subsequently create mathematical objects. It begins with marks embedded in a disciplined practice of recognition, substitution, repetition, and comparison.

The important transition occurs when marks cease to function merely as strokes on paper and become stable participants in a formal system. A circle is no longer only a drawn shape. It becomes an element that may occupy an input position, appear as an output, and be identified

across repeated occurrences. The mathematical object emerges through a rule-governed pattern of use.

2.1 Distinguishable Marks, Equality, and Repeatability

Suppose the investigator begins with three marks:

○, △, □.

These marks are not yet numbers, truth values, physical objects, or algebraic elements with familiar names. Nevertheless, they are already required to be distinguishable. The investigator must be able to tell that ○ differs from △ and that two separately written circles count as occurrences of the same mark.

This introduces an equality policy before any operation has been studied. The experiment assumes:

○ = ○,

△ = △,

□ = □,

while

○ ≠ △,

△ ≠ □,

○ ≠ □.

The equality involved is not the physical identity of ink strokes. Two circles drawn at different positions are not the same material mark. They are treated as the same symbol because the investigator recognizes them as instances of one type.

Mathematical identity therefore arises through abstraction from physical difference. Variations in size, pressure, orientation, and exact shape are ignored. A sufficiently circle-like mark is admitted as ○. A sufficiently triangle-like mark is admitted as △.

This act of recognition is already a quotienting procedure. Many distinct physical inscriptions are placed into the same symbolic class. The experiment cannot begin until this equivalence is stable enough for repeated use.

Repeatability is equally important. If the operation table assigns an output to ○ ⋄ △, the same input pair must yield the same output whenever it is consulted. Otherwise, the table does not define a deterministic operation.

The investigator therefore assumes a stable relation between inscriptions and symbolic types. Without that stability, there is no carrier set, no substitution, and no identity testing. The marks would remain transient physical events rather than mathematical elements.

2.2 Lookup Procedures as Primitive Mathematical Practice

An operation table is not self-interpreting. It must be used according to a procedure.

Suppose the row identifies the first input and the column identifies the second. To evaluate △ ⋄ □, the investigator:

1. locates the row labeled △;
2. locates the column labeled □;
3. finds their intersection;
4. reads the mark written in that cell.

The output is not discovered by visual inspection alone. It is produced through a practiced movement across an organized surface.

The table therefore functions as equipment. Its meaning lies partly in what the investigator knows how to do with it. A person unfamiliar with operation tables may see only an arrangement of marks. A mathematically trained user sees a rule for converting ordered pairs into outputs.

The lookup procedure supplies several assumptions:

- rows and columns have distinct roles;
- their intersection determines one result;
- labels remain stable across the table;
- the cell content is read as an output rather than another input;
- the procedure may be repeated without changing the rule.

These assumptions are not written in every cell. They belong to the practice in which the table is embedded.

The operation is therefore more than the visible array of marks. It includes the convention governing how the array is consulted. A transposed reading convention could produce a different operation from the same written grid.

This point becomes important when discussing anonymity. The operation may lack a familiar algebraic name, but it already possesses a practical meaning:

given two marks in specified positions, use the table to obtain a third.

That meaning precedes classification as addition, subtraction, meet, join, implication, or any other named operation.

2.3 Ordered Inputs, Closure, and Deterministic Outputs

A binary table ordinarily treats inputs as ordered. The pair (○, △) need not have the same output as (△, ○). The first position and second position are distinct even when both draw their values from the same carrier.

This distinction is already a structural commitment. The experiment does not begin with an unordered combination of two marks. It begins with a map from ordered pairs:

$A \times A \rightarrow A$.

If the carrier is

$A = \{○, △, □\}$,

then the domain consists of all ordered pairs drawn from these marks. The order matters unless later calculation proves that the operation is commutative.

Closure is also supplied in advance. Every output lies in the same carrier A. This allows the result of one lookup to become an input to another.

For example, if

○ ⋄ △ = □,

then the expression

(○ ⋄ △) ⋄ ○

can be evaluated because the intermediate result □ is again an admissible input.

Closure is not discovered from the table if every cell has already been required to contain one of the carrier marks. It is part of the table's construction.

Determinism is similarly presupposed. Each ordered pair has exactly one output. The table does not contain probability distributions, multiple possible results, or undefined cells.

Thus, before the investigator discovers a single identity, the system already includes:

- ordered argument positions;

- totality;
- closure;
- determinism;
- repeatability.

These conditions define a substantial mathematical world. The later theory emerges within that world rather than from an absence of structure.

2.4 Substitution, Iteration, and the Formation of Terms

Once outputs can be reused as inputs, simple lookups develop into terms.

The investigator may write:

x ⋄ y,

(x ⋄ y) ⋄ z,

x ⋄ (y ⋄ z),

(x ⋄ x) ⋄ (y ⋄ z).

These expressions are not additional operation-table entries. They are formal constructions generated from variables and the operation symbol.

Term formation introduces syntax. Parentheses determine evaluation order. Variables stand for arbitrary carrier elements. Repeated variables must receive the same substitution throughout one evaluation.

For the term

(x ⋄ x) ⋄ y,

if x is replaced by △ and y by □, both occurrences of x must be replaced by △. The result is:

(△ ⋄ △) ⋄ □.

The investigator must first evaluate the inner expression and then use its output in the outer expression.

Iteration therefore depends on a temporal or logical sequence. Some part of the expression is evaluated before another. The result is retained and transported into the next step.

Even when the work is done entirely on paper, memory is involved. The investigator may hold an intermediate mark mentally, write it beside the expression, or copy it into another location. The mathematical process depends on a practical ability to preserve identity across stages.

Terms enlarge the observational range of the experiment. The original table gives only the primitive operation. Terms reveal the derived operations produced through composition.

A term in two variables defines a new binary function on the carrier. A term in three variables defines a ternary function. Two different terms may produce the same derived function under every assignment.

This is where identities become visible. If

$(x \diamond y) \diamond z$

and

$x \diamond (y \diamond z)$

always yield the same output, then the operation is associative.

The law is not supplied as a target if the expressions were generated neutrally. But the possibility of discovering it depends on prior commitments to substitution, parenthesization, iteration, and equality of outputs.

2.5 The Prior-Structure Ledger

A prior-structure ledger records everything introduced before the alleged discovery occurs. Its purpose is not to invalidate the experiment. It distinguishes what was supplied from what was obtained.

For a simple table experiment, the ledger should include at least:

- the number and identity policy of the carrier marks;
- the arity of the operation;
- the ordering of argument positions;
- the requirement of one output for each input pair;
- closure of outputs within the carrier;
- the table-reading convention;
- the availability of variables;
- the rules for parenthesization;

- the permission to reuse outputs as inputs;
- the meaning of equality between outputs;
- the collection of terms selected for testing;
- the stopping rule for the investigation.

This list often reveals that much more has been supplied than the unfamiliar symbol suggests.

The investigator may also record the practical purpose of the exercise:

- search for repeated behavior;
- identify term collisions;
- compress the table into laws;
- compare the result with known algebra.

That purpose shapes attention. A person using the same table for encryption, puzzle solving, or artistic pattern generation might disclose different features.

The ledger should therefore include not only formal assumptions but the project within which the table is encountered.

A second ledger should record only what was obtained from calculation:

- constant diagonals;
- identities;
- counterexamples;
- derived operations;
- symmetries;
- classifications;
- reducibility to known structures.

The two ledgers must remain separate. An item should not migrate from the supplied list to the discovered list merely because it became less visible during the experiment.

For example, closure cannot later be claimed as an emergent law if the table format required every output to belong to the original carrier. By contrast, the existence of a two-sided identity may be a genuine discovery if no element was designated in advance for that role.

The prior-structure ledger creates an audit trail from setup to conclusion. It is the pencil-and-paper analogue of documenting software dependencies, parameter settings, and background signatures in a computational experiment.

2.6 What Pencil-and-Paper Experiments Actually Presuppose

Pencil-and-paper work avoids some assumptions built into digital computers, but it does not provide a structureless substrate.

It presupposes a human body capable of:

- making controlled marks;
- visually distinguishing them;
- preserving attention across time;
- remembering conventions;
- repeating procedures;
- detecting sameness and difference;
- following ordered steps;
- treating symbols as representatives of types.

It also presupposes a material environment in which inscriptions persist long enough to be compared. Paper retains marks. Spatial arrangement remains stable. Rows and columns can be revisited. The physical world supports repeatable interaction.

Beyond these bodily and material conditions, the experiment presupposes a mathematical culture. The investigator already understands what it means for one mark to stand for an arbitrary element, for an expression to contain a variable, and for parentheses to govern order of evaluation.

Even an attempt to avoid familiar notation uses inherited practices of formalization.

This does not make every result predetermined. A chess player knows the rules without knowing how a particular game will unfold. Likewise, the investigator may know the operation-table framework without knowing which identities the table satisfies.

The distinction is between the conditions of possibility and the outcome.

The supplied practices make discovery possible. They do not necessarily determine what will be discovered.

The pencil-and-paper experiment can therefore support a legitimate claim:

A particular law, derived operation, or structural classification was not explicitly supplied and became visible through systematic engagement with the table.

It cannot support the stronger claim:

Mathematics arose from an entirely nonmathematical starting point.

The marks were mathematical objects because they were already treated according to stable practices of identity, substitution, and use. The table was already an operation because it was already embedded in a procedure that transformed ordered inputs into repeatable outputs.

What the investigation adds is articulation. It converts operational familiarity into explicit structure.

The progression is therefore:

physical inscriptions → recognized symbolic types → rule-governed use → generated terms → detected regularities → mathematical objects and laws.

At no stage is there a leap from absolute meaninglessness. Each stage makes explicit a structure that the preceding practice already enabled.

The significance of the experiment lies precisely in this layered transition. Mathematics appears not as a creation from nothing, but as the stabilization and articulation of relations already active in a disciplined form of engagement.

3. Syntax, Semantics, and Interpretation

The distinction between an anonymous symbol and a defined operation becomes clearest when syntax and semantics are separated. Syntax concerns the formal expressions that may be written and the rules governing their construction. Semantics concerns what those expressions denote when interpreted in a particular structure. Interpretation connects the two.

A symbol such as ⋄ has very little content at the syntactic level. It may be declared binary, meaning that it combines two expressions to form a new expression. From variables x and y, one may therefore write $x \diamond y$. From that expression and another variable z, one may write $(x \diamond y) \diamond z$ or $x \diamond (y \diamond z)$.

Nothing in the symbol alone determines how any of these expressions should evaluate. The mark ⋄ does not inherently mean addition, multiplication, implication, subtraction, or

composition. It acquires operational meaning only when a structure assigns it a concrete function.

Once a table is supplied, however, the situation changes. The symbol remains formally neutral, but its interpretation is no longer indeterminate. The table assigns one output to every pair of carrier elements. The operation is then fully realized within that structure.

This separation prevents two opposite errors. The first is to mistake the symbol for the operation, as though changing notation changed the mathematics. The second is to mistake the absence of a familiar name for the absence of interpretation. The correct account is layered: the symbol may be anonymous while its realization is exact.

3.1 Function Symbols and Interpreted Operations

A formal language may contain a binary function symbol $\diamond$. At the syntactic level, the symbol is characterized by its arity. It accepts two terms and produces a new term.

If x and y are variables, then $x \diamond y$ is well formed. If t_1 and t_2 are already well-formed terms, then $t_1 \diamond t_2$ is also well formed.

This recursive rule generates expressions such as:

x,

$x \diamond y$,

$(x \diamond y) \diamond z$,

$x \diamond (y \diamond z)$,

$(x \diamond x) \diamond (y \diamond z)$.

At this stage, no outputs have been assigned. The expressions are formal arrangements of symbols.

To evaluate them, a structure must be supplied. Suppose the carrier is A. An interpretation assigns to $\diamond$ an actual binary operation:

$\diamond^A : A \times A \rightarrow A$.

The superscript indicates that the operation belongs to the structure A. The same syntactic symbol may receive different interpretations in different structures.

In one structure, $\diamond$ may be addition modulo three. In another, it may be maximum on an ordered set. In a third, it may be a projection:

$x \diamond y = x$.

The syntax remains unchanged while the semantics varies.

This distinction is fundamental to algebra. Equations are not normally statements about marks on paper. They are statements about how interpreted terms behave in structures.

When a table is introduced, it fixes one interpretation of the symbol. The symbol itself remains capable of other interpretations, but the current experiment is no longer semantically open. The table determines $\diamond^A$ exactly.

Thus the phrase “anonymous operation” combines two levels:

- the function symbol is anonymous with respect to familiar meaning;
- the interpreted operation is defined by the table.

Confusion arises when these levels are treated as though they were identical.

3.2 The Difference Between a Symbol and Its Realization

A symbol is a reusable mark governed by grammatical rules. A realization is the mathematical object assigned to that symbol in a particular structure.

The plus sign provides a familiar example. The symbol + may denote ordinary addition of integers, addition of real numbers, vector addition, addition modulo n, matrix addition, or an operation in an abstract abelian group.

These are not literally one concrete function because their domains and codomains differ. They share a structural role and notation.

Likewise, $\diamond$ may be realized by many different operation tables. The mark does not determine which table is intended.

Conversely, one operation may be represented by many symbols. Addition modulo three could be written as +, $\diamond$, *, or μ without changing its input-output behavior.

The relationship is therefore many-to-many:

one symbol may have many realizations;

one realization may have many symbolic presentations.

This has an immediate consequence for novelty. Replacing + with $\diamond$ does not produce a new operation. It only changes the presentation. If the table still realizes addition modulo three, the algebra remains the same.

A stronger change would require altering the operation itself or uncovering properties not already captured by the familiar interpretation.

The distinction also clarifies what is withheld in an anonymous-operation experiment. The realization is not withheld if the table is visible. Every value is available. What is withheld is the conventional conceptual compression that tells the investigator how the table relates to known mathematics.

The operation may therefore be behaviorally transparent and conceptually opaque.

3.3 Extensional Knowledge and Intensional Recognition

Extensional knowledge specifies what an operation does for each input. Intensional recognition identifies a rule, concept, or structural role explaining that behavior.

For a finite operation, the complete table provides extensional knowledge. One can answer every question of the form:

What is $a \diamond b$?

The answer is obtained by lookup.

Yet complete extensional knowledge may coexist with poor understanding. A nine-entry table on three elements can be fully known while its governing pattern remains unnoticed.

Intensional recognition occurs when the table is compressed into a rule such as:

$x \diamond y = y - x$ modulo 3.

The formula does more than reproduce the outputs. It connects the table to modular arithmetic, cyclic groups, inverses, and cancellation.

A law system can provide another kind of intensional recognition. The investigator may discover that the operation satisfies:

$x \diamond x = c$,

$c \diamond x = x$,

$(x \diamond y) \diamond (x \diamond z) = y \diamond z$.

These identities organize the table without necessarily giving a single numerical formula.

The distinction between extension and intension is central to the meaning of discovery. If every table value was supplied, no new extensional fact about primitive inputs has been found. What may be discovered is a compressed description, a derived operation, an identity basis, or a structural classification.

This is genuine mathematical work. A finite table contains a complete list of cases, but mathematics seeks more than lists. It seeks reusable organization.

The discovery therefore consists in moving from:

all outputs are known individually

to:

the outputs are understood through a small structural account.

The first is specification. The second is recognition.

3.4 Models as Assignments of Operational Meaning

A model provides a domain together with interpretations of the symbols in a language.

If the language contains one binary symbol $\diamond$, a model consists of:

- a nonempty carrier A;
- an operation $\diamond^A$ on A.

If the language also contains a constant symbol c, the model assigns an element c^A in A. If it contains a unary symbol n, the model assigns a function n^A from A to A.

The same formal theory may have many models. For example, the associative law

$(x \diamond y) \diamond z = x \diamond (y \diamond z)$

is satisfied by many different operations on many different carriers.

The law does not define one table. It defines a class of models.

This distinction separates the study of a single operation from the formation of a theory. A table experiment begins with one model. From its behavior, the investigator extracts candidate identities. Those identities may then define a larger class of models.

The direction is:

one interpreted structure → observed identities → candidate theory → class of models.

The movement from model to theory is not automatic. A single finite model satisfies infinitely many identities, many of which may be accidental, redundant, or too specific to characterize an interesting class.

A theory must select and organize those identities.

The model therefore supplies operational meaning, while the theory abstracts from the particular realization. It preserves what can be stated generally across structures satisfying the same laws.

This is one reason a discovered identity should be tested against additional models. An equation may hold in the original table without capturing a broad structural principle. Countermodels reveal whether the proposed law extends beyond the first realization.

3.5 Identities as Equalities of Term Functions

Every term determines a function when interpreted in a finite algebra.

Suppose t(x, y, z) is a term containing three variables. Once ⋄ is interpreted on A, the term determines a function:

$t^A : A^3 \to A$.

For every assignment of values to x, y, and z, the term produces one output.

Two different terms s and t may induce the same function:

$s^A = t^A$.

This means that for every assignment of their variables, the two terms evaluate to the same carrier element.

An identity is therefore an equality between term functions.

For example, associativity asserts that the terms

$(x \diamond y) \diamond z$

and

$x \diamond (y \diamond z)$

define the same ternary function.

The syntax trees differ. Their semantic behavior agrees.

This distinction reveals the precise content of a term collision. The collision does not mean that the two written expressions are identical. It means that the interpreted functions they define are extensionally equal in the chosen model.

For a finite carrier, this can be checked exhaustively. The investigator evaluates both terms under every assignment and compares the outputs.

The resulting identity belongs to the model's equational theory. It records one way in which syntactic difference collapses under semantic interpretation.

This collapse is mathematically important because it creates equivalence classes of expressions. Many different terms may represent the same derived operation.

The algebra therefore induces a quotient of syntax by semantic equality:

two terms are identified when they define the same function.

This quotient is not imposed merely by notation. It is determined by the behavior of the interpreted operation.

The laws of the algebra can therefore be viewed as a map of which syntactic distinctions survive and which disappear.

3.6 When Naming Adds No New Mathematics

Once an anonymous operation has been classified, a familiar name may be introduced. If the table is recognized as addition modulo three, the investigator may replace ⋄ with +.

This change can improve communication. It connects the result to an established literature, standard theorems, conventional notation, and recognized structures.

But naming alone adds no new mathematical behavior.

If

$x \diamond y = x + y \text{ modulo } 3$,

then replacing ⋄ by + does not alter the operation. Every term function, identity, model, and counterexample remains the same under the translation.

The gain is interpretive and historical, not extensional.

Naming may nevertheless have major practical consequences. Once an operation is recognized as group addition, a large body of known results becomes available. The investigator can infer consequences without recomputing them from the table. The name acts as a link into an existing mathematical world.

The name therefore adds access, not structure.

This distinction is important when assessing discovery. Recognizing that ⋄ is addition modulo three may be a genuine achievement for the investigator or machine. It is not a new theorem about modular addition.

Likewise, discovering that a table satisfies the axioms of a known quasigroup, semilattice, Boolean algebra, or Kleene algebra may validate the method without establishing historical novelty.

Naming can also conceal differences. Two operations may both be called addition while belonging to different carriers or theories. Conversely, two differently named operations may be structurally equivalent.

The correct order of inquiry is therefore:

first determine the behavior;

then extract the laws;

then compare the resulting structure with known mathematics;

finally apply a name if the identification is justified.

At the end of this process, the name should be understood as recognition of an already operative structure.

The mathematical content was present in the realization and its term behavior. The name makes that content legible within a historical community.

Thus syntax, semantics, and interpretation form three distinct layers:

- syntax provides the expressions;
- semantics provides their behavior in a structure;
- interpretation connects that behavior to concepts, theories, and names.

An anonymous operation is anonymous mainly at the third layer. Its syntax is specified, and its semantics may be completely fixed by a table. What remains open is how the operation should be understood.

4. Historical Precursors to Anonymous but Defined Operations

The distinction between an anonymous symbol and a fully specified operation did not arise with computational algebra. It belongs to a longer history in logic, axiomatics, model theory, universal algebra, and the study of functional completeness. Across these traditions, mathematicians repeatedly separated what a symbol is called from what role it plays, what an object is made of from how it is related to other objects, and what a theory says from the structures in which it is realized.

The central historical shift was away from the assumption that mathematical primitives must carry an intrinsic, intuitive meaning before reasoning can begin. A point need not first be identified with a physical location. A line need not first be understood as a drawn stroke. A

binary operation need not first be called addition or multiplication. Primitive symbols can be introduced formally, constrained through axioms or tables, and studied through the consequences of those constraints.

This development created the conceptual space in which an operation such as ⋄ can be fully defined without being historically classified. Padoa emphasized that primitive symbols can be treated as lacking independent meaning. Hilbert developed theories in which objects are determined by their structural relations. Tarski formalized the distinction between a language and its models. Birkhoff made operations and identities central to universal algebra. Post studied arbitrary truth functions through closure under composition. Clone theory generalized that process. Black-box algebra showed how a structure can be operationally available while its representation remains hidden.

These traditions do not all investigate the same problem. Some begin from axioms, some from tables, some from oracles, and some from classes of functions. Together, however, they establish that anonymity is compatible with exact mathematical definition.

4.1 Padoa and Meaningless Primitive Symbols

Alessandro Padoa was among the earliest thinkers to state clearly that the primitive symbols of a formal theory need not possess an intrinsic meaning before the theory is developed. A deductive system begins with undefined symbols, axioms, and rules of inference. The axioms constrain the possible interpretations of the symbols.

The word undefined can be misleading. It does not mean that the symbols are unusable or arbitrary in every respect. Their grammatical roles are specified. One symbol may be treated as a relation, another as an operation, another as an object constant. What remains unspecified is the independent content that the symbol is supposed to represent.

A primitive symbol receives mathematical meaning through the network of propositions in which it appears. If several different systems of objects satisfy the axioms, each system supplies an interpretation. The primitive symbols do not point to one hidden essence behind all models. Their role is determined structurally.

Padoa also developed a method for testing whether one primitive notion is definable from the others. The basic idea is comparative. If two interpretations agree on all the other primitive symbols but assign different meanings to the symbol under investigation, then that symbol is not definable from the rest. If such variation is impossible, the symbol may be implicitly determined by the remaining structure.

This is relevant to anonymous operations because it distinguishes primitive introduction from later recovery. A symbol may be admitted without a familiar name while still being constrained strongly enough that its role becomes recoverable from the system.

In the table experiment, ⋄ differs from Padoa's purely axiomatic primitives because the operation is already given extensionally. Yet the same philosophical separation is present. The mark itself has no inherited meaning, while its permissible behavior is determined by the surrounding formal arrangement.

Padoa's work therefore weakens the idea that mathematical understanding must begin with intuitive content. A theory may begin with formal roles and acquire interpretation through structural realization.

4.2 Hilbert and Implicit Definition

Hilbert's axiomatic method transformed the relation between mathematical objects and their definitions. In geometry, words such as point, line, and plane were no longer required to designate objects with an independently fixed intuitive essence. Their mathematical identity was governed by the axioms relating them.

The familiar summary is that one could replace points, lines, and planes with arbitrary objects, provided the required relations were satisfied. Whether or not that formulation captures every nuance of Hilbert's position, it expresses the structural consequence of his method: the theory depends on the pattern of relations, not on the material nature of its objects.

This is a form of implicit definition. The axioms do not necessarily give a direct formula explaining what a point is. Instead, they specify how points relate to lines, how incidence works, how order and congruence behave, and what follows from those relations.

The object becomes identifiable through its place in the system.

An anonymous operation table exhibits a related structure. The symbol ⋄ is not defined through a familiar concept such as summation. Its role is supplied by the total pattern of outputs and by the way that operation participates in larger expressions.

Hilbert's approach also reveals an important limitation. Axioms may fail to determine a structure uniquely. Different nonisomorphic models may satisfy the same theory. The primitive concepts are then not fixed as one concrete realization but only up to the distinctions expressed by the axioms.

A complete finite operation table is more determinate than an axiom set. It specifies one operation on one labeled carrier. But when identities are extracted from the table, the process

moves in the opposite direction. The single table gives rise to a theory with potentially many models.

Thus the experiment passes between two levels:

- a concrete realization specified by a table;
- an abstract structure specified through laws.

Hilbert's legacy helps clarify that the second level does not depend on discovering the hidden substance of the operation. It depends on articulating the formal relations that characterize its behavior.

4.3 Tarski and the Model-Theoretic Separation of Language and Structure

Model theory gives the most precise account of the distinction between a formal symbol and an interpreted operation.

A language contains symbols organized by type. It may contain constant symbols, relation symbols, and function symbols of specified arities. At this level, a binary symbol such as $\diamond$ is part of the syntax. It can be used to construct terms, but it does not yet determine a concrete output for any pair of objects.

A structure interprets the language. It supplies a carrier and assigns a mathematical realization to each symbol. A binary function symbol receives an actual operation on the carrier.

The distinction can be written schematically as:

formal symbol $\diamond \rightarrow$ interpreted operation $\diamond^{A}$.

The first belongs to the language. The second belongs to a model A.

This separation permits one formal statement to be evaluated in many structures. The equation

$(x \diamond y) \diamond z = x \diamond (y \diamond z)$

is a syntactic object. Whether it is true depends on how $\diamond$ is interpreted.

In one structure, it may hold. In another, it may fail. The equation becomes an identity of a class of structures when it holds under every assignment in every member of that class.

Tarski's framework makes clear that a table does not merely suggest an interpretation. It supplies one. Once the table is fixed, the meaning of $\diamond$ inside that model is exact.

What remains open is not the semantic value of individual applications but the theory of the operation:

- which formulas are true in the model;
- which identities hold;
- which other structures satisfy the same sentences;
- which aspects of the operation are expressible in the chosen language;
- which properties require an expanded vocabulary.

Model theory also shows that observation is language-relative. Two structures may be distinguishable in a richer language and indistinguishable in a poorer one. If the language contains only $\diamond$, then any feature not definable from $\diamond$ remains formally invisible.

This insight applies directly to operation-first discovery. The theory that emerges is not simply the whole truth of the table. It is the truth that can be expressed through the selected language of terms, equations, constants, relations, or quantifiers.

Anonymity is therefore accompanied by a hidden restriction: the system sees the operation only through the language provided.

4.4 Birkhoff and the Operational Foundations of Universal Algebra

Garrett Birkhoff supplied the closest mathematical framework to the study of anonymous finite operations through term identities. Universal algebra treats algebraic structures in terms of carriers, primitive operations, terms, homomorphisms, subalgebras, products, quotients, and equations.

A primitive operation does not need a familiar interpretation. It may be represented by a symbol of fixed arity. From that symbol and a collection of variables, formal terms are generated recursively.

Once the symbol is interpreted in an algebra, each term becomes a derived operation. Two terms determine an identity when they agree under every assignment of variables.

This is exactly the mathematical form of the term-collision experiment.

A table supplies a finite algebra A. Terms s and t are generated. Their induced functions s^A and t^A are evaluated. If they are equal as functions, then A satisfies the equation:

$s = t$.

The totality of such equations forms the equational theory of the algebra.

Birkhoff's work also established the deep connection between equations and classes of algebras. A class definable by identities is closed under homomorphic images, subalgebras, and

direct products. Conversely, classes with the relevant closure properties can be characterized equationally.

This means that identities extracted from one or more operation tables can be viewed not merely as local observations but as generators of a larger algebraic world.

However, the transition must be handled carefully. The complete identity theory of one finite algebra may be highly specific. A short bounded list of observed collisions need not characterize an important variety. The challenge is to identify a compact, informative basis and determine what class of models it defines.

Birkhoff's framework therefore contains both the power and the limitation of operation-first discovery. The machinery for moving from operations to equations is classical. The possible novelty lies in how candidate laws are selected, organized, compared, and used to generate new classifications.

From this perspective, the phrase anonymous operation is not mysterious. It means that the operation symbol carries no conventional name while the algebra interpreting it is fully available for term evaluation.

4.5 Post, Functional Closure, and Arbitrary Truth Operations

Emil Post studied systems generated from arbitrary truth functions on a two-element domain. Instead of assuming that the primitive connectives must be familiar operations such as AND, OR, or NOT, one can begin with any truth table and ask what functions become expressible through composition.

This is a strongly operation-first perspective.

A primitive truth operation is given extensionally by a table. From it, more complicated formulas are constructed. Each formula defines another truth function. The collection of all functions obtainable from the primitives forms a closed system.

The important question is not initially what the primitive operation should be called. It is what expressive world it generates.

A single operation may turn out to be functionally complete, meaning that every truth function can be constructed from it. Other primitives generate restricted families. Different starting tables may produce the same closure even when their surface definitions differ.

This introduces a distinction between primitive presentation and generated capacity. Two operation sets may look different while generating identical functional universes. Conversely, one small change in the primitive set may radically alter what is expressible.

Post's work is therefore especially relevant to the study of background dependence. Whether a pair of operations is fundamental depends on which other operations are supplied freely. A primitive may appear essential in one background and redundant in another.

The same issue appeared in the later refactoring experiments on Boolean and Kleene structures. When negation was supplied as a free background operation, no irreducibly cooperative binary pairs appeared in the tested cases. When free negation was removed, cooperative pairs emerged.

Post's perspective shows that this is not an accidental implementation detail. It reflects a general principle:

expressive power is relative to the available primitive basis.

The operation is anonymous in name but not isolated from its environment. Its significance depends on the closure it generates together with the rest of the signature.

4.6 Clone Theory and Generated Operational Worlds

Clone theory generalizes the study of functional closure. A clone is a collection of finitary operations on a fixed set that contains all projections and is closed under composition.

Given a set of primitive operations, one may form every operation obtainable by substituting outputs of some operations into inputs of others. The resulting collection is the clone generated by the primitives.

This formalizes the entire derived-operation world of a signature.

If $\diamond$ is a binary operation on A, then terms built from $\diamond$ and variables define operations of various arities. The set of all such term operations is the clone generated by $\diamond$.

Two different primitive operations may generate the same clone. In that case, they have the same total expressive power through composition, even though their tables differ.

This fact complicates claims of novelty based on unfamiliar primitive operations. A strange-looking table may generate a well-known clone. The primitive presentation is new only superficially if its derived operational universe is already classified.

Clone theory therefore supplies a demanding recognition audit. One should ask:

- What operations are generated?
- Which projections or constants are definable?
- Is the generated clone already known?

- Does the operation belong to a classified interval of clone space?
- Does a proposed new basis generate anything structurally new?

The theory also emphasizes that higher-level algebraic organization can emerge from lower-level operations without the higher organization being explicitly listed. The clone is not written into the primitive table entry by entry. It unfolds through closure under composition.

Yet the closure rules are supplied in advance. The system already knows how outputs may be substituted into inputs and how variables may be reused or ignored.

Thus clone theory supports a relative emergence claim:

the generated operational world is not explicitly enumerated in the primitive data, but it is determined by the primitive operations together with the pre-existing rules of composition.

This is a more precise formulation than saying that algebra arises from nothing.

4.7 Black-Box Algebra and Hidden Representations

Black-box algebra studies structures whose elements and operations are accessible through procedures while their internal representation remains hidden or opaque.

In a black-box group, for example, elements may be encoded as bit strings or tokens. The investigator can request products, inverses, and equality tests without being given a transparent description of the group.

The operation is therefore operationally available. One can use it, compose it, and test consequences. What is missing is a recognizable representation.

The research task is often one of identification or reconstruction:

- determine the group's isomorphism type;
- recover a standard representation;
- identify generators;
- compute structural invariants;
- distinguish among candidate models.

This is closely related to the anonymous-table experiment, although the assumptions differ. In black-box group theory, one is normally told that the hidden operation satisfies group laws. The anonymity concerns representation, not algebraic species.

In an operation-first table experiment, even the law profile may be unknown. The investigator does not assume associativity, an identity, or inverses.

Still, the comparison is illuminating. In both settings, the structure can be used before it is recognized.

A black-box operation is not undefined. It is available through an oracle or procedure. The user knows how to request outputs. The theoretical task is to infer the larger structure from interaction.

This suggests a hierarchy of anonymity:

- notational anonymity: the familiar symbol is withheld;
- classificatory anonymity: the algebraic species is unknown;
- representational anonymity: the internal encoding is hidden;
- theoretical anonymity: the governing laws have not been articulated;
- historical anonymity: the relation to existing mathematics has not been recognized.

These forms should not be conflated. An operation may be anonymous in one sense and fully known in another.

The history from Padoa through black-box algebra therefore converges on a common conclusion. Mathematical objects need not arrive with intrinsic names or transparent representations. Their roles can be determined through axioms, interpretations, tables, closure systems, or operational access.

What operation-first discovery adds is not the invention of this distinction. Its opportunity lies in making the passage from operational access to structural articulation explicit, reproducible, and increasingly automated.

5. Heidegger's Transformation of the Problem

The problem of the anonymous operation can be framed in a conventional epistemological way: an operation exists, but the observer does not yet know what it is. The table provides appearances, while the true mathematical nature of the operation remains to be discovered behind them. This framing resembles a distinction between what is given and what exists in itself.

Heidegger transforms the problem by challenging the assumption that human beings first encounter isolated objects as neutral data and only afterward attach meaning to them. Human

existence is not initially detached observation. It is being-in-the-world: practical involvement in a field of purposes, habits, tools, expectations, and concerns.

From this perspective, the operation ⋄ is never first encountered as a meaningless object waiting for interpretation. It appears within an activity. The investigator knows how to consult its table, substitute marks, preserve intermediate results, compare outputs, and search for regularities. Even before the operation receives a familiar mathematical name, it already has a practical role.

The philosophical question therefore changes. It is no longer primarily:

What hidden mathematical thing lies behind the table?

It becomes:

How does an operation already functioning within a practice become explicitly visible as a mathematical object?

This shift is decisive for the interpretation of Algebraic Abiogenesis. The process does not move from complete meaninglessness to algebra. It moves from implicit operational involvement to explicit theoretical articulation.

5.1 Beyond the Thing-in-Itself

Kant distinguished between phenomena, things as they appear under the conditions of human cognition, and things-in-themselves, whose nature independent of those conditions cannot be directly known. The distinction was intended to mark the limits of human knowledge.

A simplified application of this framework to the anonymous operation might suggest that the table presents only an appearance, while the operation's true nature remains hidden behind it. The investigator observes outputs and attempts to infer what ⋄ really is.

Heidegger resists this picture when it is taken to imply that ordinary experience confronts only a surface representation standing between the observer and the real world. Human beings do not ordinarily begin as detached subjects receiving internal appearances of external objects. They are already involved in a world.

The table is therefore not merely a veil concealing a deeper operation. The table is one mode in which the operation is available. The operation is encountered through the practice of consulting the table.

If the table completely specifies every value of ⋄, there is no second operational object hidden behind it. A later formula such as modular subtraction may provide a more illuminating

interpretation, but it does not reveal an inaccessible thing-in-itself. It reorganizes and compresses the same operative behavior.

The mystery lies not behind the table but in the transition from use to understanding.

5.2 Dasein as Being-in-the-World

Heidegger uses the term Dasein for the kind of being whose own existence and world can matter to it. Dasein is not first an isolated mind that subsequently enters into relations with external objects. Its basic condition is being-in-the-world.

The world in this sense is not merely the total collection of physical objects. It is an organized context of significance. Things appear as usable, obstructive, relevant, dangerous, familiar, or unnecessary within ongoing projects.

A hammer appears as something for driving nails. A door appears as something to be opened, closed, entered through, or blocked. Their significance does not arise only after a neutral sensory object has been theoretically identified.

Likewise, an operation table appears as something for obtaining outputs, testing substitutions, and investigating patterns. The investigator encounters it within the project of mathematical inquiry.

This project supplies a horizon of significance:

- outputs should remain stable;
- repeated terms should be compared;
- coincidences may indicate laws;
- counterexamples may defeat generalizations;
- compact descriptions are preferable to exhaustive lists;
- known classifications matter.

These expectations belong to the world in which $\diamond$ becomes intelligible.

The operation is therefore not anonymous in every sense. It is anonymous with respect to conventional algebraic naming, but not with respect to its use in an investigative practice.

5.3 Practical Involvement Before Theoretical Observation

Heidegger argues that practical involvement is more basic than detached theoretical observation. We usually encounter things by using them rather than by first cataloging their objective properties.

A craftsperson does not ordinarily begin by contemplating the hammer as an object with mass, shape, material composition, and spatial coordinates. The hammer is encountered in hammering. Its properties become explicit when they matter to the task.

The same pattern applies to an operation table. The investigator does not first receive $\diamond$ as a fully articulated theoretical object. The operation is used:

- a row is selected;
- a column is selected;
- an output is read;
- the output is carried into another expression;
- two results are compared.

Through this practical sequence, the operation is already understood in a limited but genuine way. The investigator understands what to do with it.

This understanding need not include a definition in words. It may be embodied in competent performance. A person may correctly calculate complex terms without yet knowing that the operation is associative or group-derived.

Theoretical observation arises when the practice itself becomes an object of attention. The investigator stops merely calculating and asks:

Why do these terms keep agreeing?

What law explains this repeated pattern?

What kind of operation behaves this way?

The explicit mathematical object appears through reflection on a practice already underway.

5.4 Readiness-to-Hand and Operational Use

Heidegger describes equipment in its ordinary practical mode as ready-to-hand. A tool is most fully available when it is absorbed into skilled activity and does not demand separate attention.

When the table is used fluently, the investigator may no longer notice the mechanics of lookup. Attention passes through the table toward the result. Rows, columns, and symbols function transparently.

The operation is then ready-to-hand in a mathematical sense. It is something used to transform expressions.

This mode of availability explains why much prior structure can remain invisible. The investigator may overlook:

- the convention assigning first input to rows;
- the stable identity of the marks;
- the totality of the table;
- the reuse of outputs;
- the ordering imposed by parentheses.

These conditions disappear into successful use. They become noticeable mainly when something goes wrong.

The anonymity of $\diamond$ is therefore partly produced by transparency. Because the operation works smoothly as equipment, its inherited mathematical scaffolding recedes from view. The unfamiliar symbol seems neutral precisely because the surrounding practice is familiar.

The prior-structure ledger attempts to reverse this concealment. It makes explicit the practical conditions normally absorbed into use.

5.5 Presence-at-Hand and Mathematical Objectification

Heidegger contrasts readiness-to-hand with presence-at-hand. In the latter mode, something is considered as an object possessing explicit properties.

The distinction is not absolute. Practical equipment can become present-at-hand when it is examined, measured, theorized, or removed from ordinary use.

The operation $\diamond$ becomes present-at-hand when the investigator begins to characterize it explicitly:

- it is binary;
- it is total;
- it is noncommutative;
- it has a left identity;
- its diagonal is constant;
- it satisfies a particular term identity.

The operation is now treated as an object of mathematical description rather than merely as a functioning rule.

This objectification is necessary for formal mathematics. A theorem requires explicit statements, definitions, and conditions. The operation must become describable independently of a single act of lookup.

Yet Heidegger's analysis suggests that this theoretical object is derived from practical engagement. The investigator first uses the table and only later thematizes the operation.

This does not make formalization less rigorous. It changes the account of how formal objects become available. They are not necessarily presented first as neutral entities. They are extracted from a prior field of meaningful activity.

The movement can be described as:

operational use → interruption or pattern → explicit attention → formal property → mathematical object.

5.6 Why the Neutral Operation Is Already Embedded in Meaning

The ideal of a completely neutral operation assumes that one can strip away every interpretation and retain only bare data. Heidegger's analysis shows why this is impossible.

Even the attempt at neutrality belongs to a project. The investigator has chosen to suppress familiar names in order to reduce bias and test what structure can be recovered. That purpose gives the anonymous symbol its role.

The operation is meaningful as:

- something to be evaluated;
- something whose outputs can be compared;
- something that may conceal a law;
- something that may belong to a known theory;
- something whose novelty matters.

This practical meaning precedes any classification as addition, subtraction, conjunction, or another named operation.

The table is also embedded in a broader referential network. Variables refer to carrier elements. Parentheses refer to evaluation order. Equality refers to repeated output identity. A collision refers to a possible law. A law refers to a theory. A theory refers to mathematical history and future use.

No item has meaning in isolation. Each becomes intelligible through its relations to the others.

Therefore, the operation is not first meaningless and later meaningful. Its meaning changes form.

Initially, its meaning is operational and practical:

use this table to obtain a result.

Later, its meaning becomes structural:

this operation satisfies these laws.

Finally, it may become historical and theoretical:

this is a presentation of a known quasigroup, group-derived operation, clone, or variety.

The movement is not from no meaning to meaning. It is from one mode of intelligibility to another.

For Algebraic Abiogenesis, this leads to a necessary reinterpretation. The process cannot literally describe the creation of algebra from a non-algebraic void. The investigator, table, symbols, and procedures already belong to a mathematical world.

What the process can describe is disclosure:

an operation already available in practice becomes explicit as a structured object of theory.

The operation was anonymous only relative to the name it had not yet received. It was never anonymous relative to the world in which it was being used.

6. The Operation Table as Equipment

An operation table is usually described as a representation of a function. From a formal perspective, this is correct: each ordered pair of carrier elements is assigned one output. Yet this description captures only the table's abstract structure. It does not describe how the table is actually encountered and used.

In practice, an operation table functions as equipment. It belongs to an activity. A person selects an input, finds a row, finds a column, locates their intersection, reads an output, and carries that output into another calculation. The table is not merely observed. It is operated.

This practical character matters because it shows that mathematical meaning does not begin only after a law has been stated. The table already has a role before its algebraic classification is known. It is something by which results are obtained, expressions are evaluated, and regularities are tested.

The operation is therefore available in two different modes. At first, it functions transparently within calculation. Later, when a pattern, failure, or unexpected collision draws attention, the operation becomes an explicit object of investigation. The investigator no longer merely uses ⋄. The investigator begins asking what ⋄ is.

The transition from equipment to object is one of the central events in mathematical disclosure.

6.1 The Table Within a Practical Context

A table does not carry its full meaning in isolation. It belongs to a context of use.

The grid, row labels, column labels, and cell entries become mathematically intelligible only because the investigator approaches them with a purpose. The purpose may be to evaluate expressions, classify an operation, search for laws, or test whether two terms agree.

The same arrangement of marks could be interpreted differently in another practice. It might be a game board, a coded message, a pattern generator, or a decorative array. What makes it an operation table is not only its visual form but the convention according to which it is used.

The practical context includes expectations:

- each input pair has one result;
- the same lookup can be repeated;
- outputs remain stable;
- the row and column roles do not change;
- outputs may be reused in later calculations;
- repeated agreement may be mathematically significant.

These expectations are not discovered from the table one at a time. They are part of the activity that makes the table usable.

The table therefore belongs to a referential network. Its rows refer to first inputs. Its columns refer to second inputs. Its cells refer to outputs. Its outputs refer to possible later inputs. Its repeated patterns refer to possible laws.

The operation becomes meaningful through this network before it receives any conventional algebraic name.

6.2 Rows, Columns, Marks, and Embodied Procedures

Using an operation table requires bodily and perceptual activity.

The eye follows a row. The hand may trace a column. The investigator holds one input in memory while locating another. A result is copied into a new expression. Parentheses guide the order in which the hand performs successive lookups.

These actions are so familiar that they usually disappear from attention. Yet they are part of the mathematical event.

Suppose one evaluates:

$(x \diamond y) \diamond z$.

The procedure is not instantaneous. One must first determine $x \diamond y$. The result must then be retained. That result becomes the first input of a second lookup, while z becomes the second.

The expression therefore coordinates several embodied capacities:

- visual discrimination;
- spatial navigation;
- short-term memory;
- sequencing;
- copying;
- checking;
- correction.

The operation table is usable because these capacities cooperate with the stable arrangement on paper.

This does not reduce mathematics to psychology. The law discovered is not merely a mental event. But the route by which it becomes accessible is mediated by embodied practice.

The table is therefore not a neutral object placed before an abstract mind. It is a device integrated into a procedure performed by a situated investigator.

6.3 The Operation as Something Used Before It Is Theorized

An investigator can use an operation correctly without knowing its algebraic classification.

One may evaluate hundreds of expressions with $\diamond$ while remaining unaware that the operation is associative, idempotent, group-derived, or functionally complete.

This is similar to using a familiar tool without being able to explain its engineering. Competent use does not require explicit theory.

The operation first appears through what can be done with it:

- combine two marks;
- obtain a third;
- repeat the process;
- compare results;
- build more complex expressions.

Only later does the investigator step back and ask what properties make those procedures behave as they do.

This order is important. It means that the theoretical object is not necessarily primary. The operation is first encountered through practical availability.

The relation can be expressed as:

use → repetition → pattern → reflection → theory.

The operation is not initially absent and later created. Nor is it initially known in full conceptual detail. It occupies an intermediate condition: operationally available but theoretically unarticulated.

That condition is more accurate than either “meaningless” or “fully understood.”

6.4 Breakdown, Surprise, and the Appearance of Explicit Structure

Equipment becomes conspicuous when ordinary use is interrupted.

A hammer becomes noticeable when it breaks. A door becomes noticeable when it jams. An operation table becomes noticeable when an expected pattern fails or an unexpected pattern appears.

Suppose the investigator assumes, perhaps unconsciously, that changing the input order should not matter. Then one finds:

$a \diamond b \neq b \diamond a$.

The failure makes the distinction between argument positions explicit. Noncommutativity becomes visible because an expectation was violated.

Conversely, suppose two complicated expressions repeatedly produce the same output. The agreement may interrupt ordinary calculation because it appears too persistent to be accidental.

The investigator then stops merely using the operation and begins examining it.

Breakdown and surprise therefore have related roles:

- breakdown reveals a hidden dependency;
- surprise reveals a hidden regularity.

A failed conjecture discloses the limits of a theory. A persistent collision discloses a possible law. In both cases, something previously absorbed into practical use becomes an explicit object of attention.

This is why counterexamples are mathematically productive. They do more than reject statements. They expose which distinctions the original understanding failed to preserve.

Theoretical attention is often born from interruption.

6.5 Term Collisions as Moments of Disclosure

A term collision occurs when two syntactically different expressions produce the same output under every permitted assignment.

For example:

$(x \diamond y) \diamond z$

and

$x \diamond (y \diamond z)$

may differ visibly while defining the same term function.

At first, each expression is simply a procedure for using the table. One begins with inputs, performs lookups in a specified order, and obtains an output.

When the two procedures repeatedly converge, their agreement becomes significant. The investigator no longer sees only two calculations. The investigator sees one underlying behavior expressed through two forms.

The collision discloses an equivalence.

This moment transforms the character of the operation. What was previously a sequence of lookups becomes evidence of law. The table begins to appear not as an arbitrary list of outputs but as a structured whole.

The law is not added externally to the operation. It articulates a regularity already operative in repeated use.

Yet disclosure remains relative to the chosen terms. If the two expressions are never generated or compared, the law may remain unnoticed. The operation satisfies the identity whether or not anyone observes it, but the identity becomes mathematically explicit only through a particular practice of comparison.

Term generation therefore defines a horizon of visibility. Some laws can appear within it; others cannot.

A collision is thus both objective and situated:

- objective because the equality holds for all assignments in the table;
- situated because it becomes visible only through selected expressions, procedures, and criteria of comparison.

6.6 From Transparent Use to Thematic Mathematical Attention

The transition from use to theory does not require abandoning practice. It requires redirecting attention toward the structure that practice has been relying on.

At first, the investigator asks:

What output does this expression produce?

Later, the question becomes:

Why do these expressions always agree?

Then:

What law explains that agreement?

Finally:

What kind of algebra is characterized by that law?

Each question moves farther from the immediate act of lookup and toward thematic mathematical attention.

The operation becomes an explicit object possessing properties, consequences, and historical relations. It can be compared with other operations, placed within a class, described by axioms, and studied independently of the original table.

This movement can be summarized as:

transparent operation → noticed pattern → articulated identity → classified structure.

The table has not changed. The investigator's mode of access has changed.

This is the central meaning of mathematical disclosure in the operation-first setting. Algebra does not emerge because an undefined mark somehow acquires structure. It emerges because the structure already enacted in use becomes visible, stable, and expressible as law.

The table first functions as equipment. It later becomes evidence. Finally, it becomes a model of a theory.

The operation was present throughout. What changed was the level at which its organization became available to thought.

7. Addition, Combination, and the World

Addition is often treated as the most elementary mathematical operation. Two things are combined with two more things, and the result is four. Because this pattern is familiar, it is tempting to regard addition as transparently present in the world before any symbolic description is introduced.

The situation is more complicated. Physical combination, numerical addition, and conceptual representation are related but not identical. Two bodies may be brought together without any measured quantity adding exactly. Two liquids may occupy less volume after mixing than the sum of their separate volumes. Energy may leave a system as heat. Objects may merge, deform, react, or lose their former identities.

Addition therefore does not follow merely from the fact that two things become one collection or system. A quantity is additive only when the value assigned to a composite is equal to the sum of the values assigned to its parts under specified conditions.

This distinction matters philosophically. The world may instantiate additive relations without writing equations or possessing concepts. A human observer may recognize those relations, select the relevant quantities, and formalize them using the sign +. The symbol does not create the structure, but neither is every physical act of combination automatically an instance of arithmetic addition.

The question is not simply whether addition exists in the world. It is how physical composition, measurement, abstraction, and conceptual recognition cooperate so that something becomes available as addition.

7.1 Physical Combination and Additive Structure

Suppose two containers of liquid are poured into one vessel. At the most immediate level, two separated portions have become one combined portion. This is physical composition.

The mathematical question is whether some quantity Q satisfies:

Q(combined system) = Q(first part) + Q(second part).

If it does, Q is additive under that mode of combination.

The operation of physically joining systems and the operation of numerically adding measurements should therefore be distinguished. One is an event in the world. The other is a relation among assigned quantities.

The distinction can be represented as:

physical parts → physical combination → measured quantities → additive relation.

A physical event may support several measurements. Mass, volume, energy, charge, particle number, and entropy may behave differently under the same act of combination.

Some quantities may add exactly. Others may add only approximately. Others may fail to add at all.

This means that addition is not a property of combination in general. It is a property of a quantity relative to a composition rule and a domain of conditions.

An additive structure emerges when a method of combining systems and a method of assigning values are compatible.

7.2 The Difference Between Composition and Numerical Addition

Composition means that components are brought into some larger arrangement. Addition means that a numerical or algebraic relation of a particular form holds.

The two can coincide, but they need not.

Consider two pieces of clay. When pressed together, the combined object contains both pieces. If no material is lost, its mass is approximately the sum of their masses. Its surface area, however, is usually less than the sum of their original surface areas because some exposed surfaces disappear at the interface.

The same physical composition therefore supports one additive quantity and one nonadditive quantity.

A further distinction concerns identity. Suppose two drops merge into one drop. Before the merger, there are two droplets. Afterward, there is one. Counting droplets is not additive across the transformation:

2 droplets → 1 droplet.

Yet the number of water molecules may remain essentially conserved.

The result depends on what is being counted.

Addition requires a policy for determining units. If the units change during combination, naive counting fails. A collection of two systems does not guarantee that every property of those systems adds.

Thus, when one says that combining two volumes is addition, the statement is incomplete. It becomes correct only after specifying the quantity, the composition, and the physical conditions under which additivity holds.

7.3 When Measurable Quantities Are Additive

A measurable quantity is additive when the value of a composite can be obtained by summing the values of disjoint components.

For a quantity Q and two compatible systems A and B:

Q(A combined with B) = Q(A) + Q(B).

This relation depends on several assumptions.

The systems must be identified as parts of a composite. The measuring procedure must apply consistently before and after combination. No relevant quantity may enter or leave the system. Interactions must not create a correction term that the simple sum ignores.

In many practical settings, additivity is approximate rather than exact. Small losses, measurement error, thermal expansion, chemical reaction, or boundary effects may alter the result.

The mathematical operation + is exact within the formal theory. Its physical application is mediated by a model.

The model asserts that a certain measured quantity behaves sufficiently like an additive homomorphism from physical composition to a numerical structure.

The physical relation can be written schematically as:

physical composition → numerical addition.

This mapping is not automatic. It is an empirical and theoretical achievement. The observer discovers that a selected quantity respects the composition law within a specified range of conditions.

Addition in applied mathematics therefore involves both world and representation. The world supplies regularity. The observer supplies a measurement scheme and a formal language capable of expressing it.

7.4 Volume Contraction and the Failure of Naive Addition

Volume provides a clear example of why physical combination must not be identified automatically with addition.

If one volume V_1 and another volume V_2 are combined, one might expect:

$V_{combined} = V_1 + V_2$.

For many idealized systems, this approximation is useful. But real liquids can contract or expand when mixed. Molecular interactions allow the combined material to occupy a volume different from the simple sum of its initial volumes.

The more accurate relation may be written as:

$V_{combined} = V_1 + V_2 + \Delta V_{mix}$,

where ΔV_{mix} is the volume change caused by mixing.

If ΔV_{mix} is negative, contraction occurs. If it is positive, expansion occurs.

The existence of the correction term shows that the physical process is not represented by simple addition alone.

This does not imply that arithmetic has failed. It implies that the chosen physical model was incomplete.

The observer must determine which variables and interaction terms are required to describe the system. The mathematical structure may involve addition, but addition is embedded in a richer law.

This example also shows why the world cannot be reduced to a collection of obvious arithmetic acts. What appears to be “one plus one” may conceal interaction, transformation, and emergent structure.

Addition is present, but not always alone.

7.5 Mass Conservation and Physical Summation

Mass often provides a stronger example of additivity than volume, particularly in ordinary closed systems where no material is lost.

If two bodies with masses m_1 and m_2 are combined, one ordinarily writes:

$m_{combined} = m_1 + m_2$.

This works because mass is treated as conserved under the process and because the measurement scale remains consistent.

Even here, qualifications are necessary. At sufficiently high precision, energy exchange and relativistic effects complicate the simple relation. Binding energy can affect the mass of a composite system. Material may escape as vapor, radiation, or fragments.

The practical equation remains highly successful because the correction is negligible in most ordinary situations.

The lesson is not that addition is merely subjective. The success of mass addition is constrained by the behavior of the world. An observer cannot arbitrarily decide that mass will multiply when two bodies are joined.

At the same time, the additive law becomes explicit only through a conceptual framework involving:

- distinguishable systems;
- measurable mass;
- conservation;
- composition;
- numerical representation.

The world supports the relation, but the formal statement depends on a mode of disclosure.

The additive structure is neither a free invention nor a self-announcing inscription in nature. It is a stable correspondence between physical behavior and mathematical articulation.

7.6 Does the Universe Instantiate Addition Without Knowing It?

A universe without human beings could still contain systems whose measurable properties obey additive relations.

Particles could combine. Charges could sum. Conserved quantities could be distributed among subsystems. Repeated processes could accumulate effects.

In this sense, the universe could instantiate addition.

Whether it could know addition is a different question.

Knowledge ordinarily implies that a relation is represented, recognized, or available to some form of awareness. A physical system may behave according to a law without formulating that law.

A falling object does not need to know gravitational theory in order to fall. Two charges do not need to know arithmetic in order for total charge to be additive.

The distinction is:

- instantiation: the world exhibits a structure;
- representation: a system encodes or tracks the structure;
- knowledge: a knower recognizes the structure as such;
- notation: a community expresses it using symbols such as +.

A humanless universe may possess the first without the remaining three.

One could adopt a metaphysical view in which the universe itself is mental or proto-conscious. Under such a view, cosmic knowledge might be meaningful. But that claim goes beyond what algebra or physical measurement can establish.

The restrained conclusion is that the universe can realize additive structure without being described as knowing addition.

The verb know should not be used merely because a system behaves regularly.

7.7 Structure, Representation, and the Necessity of a Knower

The relation between world and mathematics can be organized into three layers.

The first is structure. Physical systems combine, separate, persist, transform, and exhibit regularities.

The second is representation. A cognitive or computational system assigns marks, numbers, variables, and operations to those regularities.

The third is recognition. A knower identifies a stable relation as an instance of addition and places it within a broader mathematical theory.

The knower does not create the physical regularity. But the knower is necessary for the regularity to appear as the explicitly articulated operation +.

This does not imply that mathematics is purely subjective. The world constrains successful representations. A false additive model eventually encounters counterexamples.

Nor does it imply that addition exists only as ink or mental imagery. The structural relation may be instantiated independently of human awareness.

The decisive distinction is between structure and structure-as-known.

The world may contain additive relations. Mathematics is the disciplined articulation of those relations through concepts, symbols, proofs, and systems of inference.

For the philosophy of anonymous operations, this means that the table does not need to conceal a mysterious operation behind its entries. The operation is present in the pattern of use. The human observer makes that pattern explicit as law.

Similarly, the universe need not contain a written plus sign in order to support addition. But the plus sign, the general law, and the concept of additivity belong to a practice of representation and recognition.

The movement is therefore:

physical combination → stable quantitative relation → recognition of additivity → formal operation +.

This sequence avoids two extremes.

It avoids the claim that humans arbitrarily invent addition without constraint from the world.

It also avoids the claim that nature literally performs symbolic arithmetic in the same sense that a mathematician does.

Addition is disclosed where physical regularity and conceptual articulation meet.

The universe supplies relations. The knower supplies explicit mathematics.

8. Why Algebraic Abiogenesis Cannot Be Absolute

The phrase Algebraic Abiogenesis suggests the birth of algebra from something that is not yet algebraic. Taken literally, however, this is difficult to defend. Every experiment designed to reveal an operation, law, identity, or theory already relies on distinctions, transformations, memory, comparison, and repetition. These are not neutral conditions. They are forms of structure.

A finite operation table presupposes a carrier, ordered inputs, stable outputs, equality, and closure. A computer implementation presupposes logic gates, state transitions, memory addresses, instruction sequences, and data representations. A pencil-and-paper implementation

presupposes a human observer capable of recognizing marks, following procedures, and preserving identity across time.

At every level, the supposed beginning is preceded by another layer of organization.

The problem is therefore not merely that the experiment contains too much prior mathematics. The deeper problem is that a completely structureless starting point would be incapable of supporting any experiment at all. Without difference, nothing could be distinguished. Without persistence, no result could be compared with another. Without transformation, no operation could occur. Without identity, no regularity could be recognized.

Absolute abiogenesis of algebra would require algebra to arise from a condition that lacks even the prerequisites for appearance, comparison, and relation. Such a condition is not experimentally accessible and may not be conceptually coherent.

What remains possible is relative emergence. A higher-level algebraic organization may become explicit within a lower-level substrate that does not contain that particular theory as a named, privileged, or targeted structure.

The distinction is therefore:

absolute birth from nonstructure → incoherent or inaccessible;

relative emergence from prior structure → coherent and testable.

8.1 The Impossibility of a Structureless Starting Point

A structureless starting point would contain no distinguishable elements, no relations, no transformations, and no persistence.

If there are no distinguishable states, then no input can differ from another. If there are no relations, then nothing can be combined, ordered, compared, or transformed. If there is no persistence, then an earlier condition cannot be retained long enough to be related to a later one.

Under such conditions, neither mathematics nor experiment can begin.

Even the assertion that a structureless state exists introduces a distinction between that state and something else. The description already imposes conceptual structure.

This creates a limit on all origin narratives. One may describe the emergence of one organization from another, but the transition from absolute nonstructure to structure cannot be represented without using the very distinctions whose origin is being questioned.

In the context of algebra, the difficulty is immediate. An operation requires inputs, outputs, and a rule relating them. If none of these are available, there is no operation to study.

Thus, Algebraic Abiogenesis cannot mean:

nothing → operation → law → theory.

The earliest experimentally meaningful stage is already something like:

distinguishable states + permitted transitions + stable comparison.

That is not yet a group, ring, lattice, or field. But it is already enough structure to support algebraic description.

The project must therefore begin from a lower algebraic or proto-algebraic substrate rather than from non-algebra.

8.2 Algebra Already Present in Hardware and Software

A digital computer appears to offer a mechanical environment in which mathematical discovery can occur independently of human intuition. Yet the computer is saturated with formal structure before any discovery program is loaded.

At the hardware level, the machine contains:

- binary states;
- logical gates;
- signal composition;
- state storage;
- controlled transitions;
- synchronization;
- copying;
- conditional routing.

These components already implement Boolean and finite-state operations.

At the processor level, the machine typically contains:

- addition and subtraction circuits;
- comparison instructions;

- registers;
- program counters;
- memory addressing;
- branching;
- repeated instruction cycles.

At the software level, further structure appears:

- integers;
- arrays;
- functions;
- loops;
- recursion;
- equality tests;
- data types;
- sorting and hashing;
- file systems;
- symbolic representations.

An experiment may avoid writing the symbol +, but it may still depend on address incrementing, loop counters, array indexing, and arithmetic used by the compiler or runtime.

This means that a conventional computer cannot serve as a substrate that does not already know addition in some operational sense. Addition-like mechanisms are built into its architecture.

Even a NAND-only implementation does not eliminate prior algebra. NAND is itself a binary Boolean operation. Gates compose. Signals persist. Wires preserve and transmit distinctions. Flip-flops implement state.

The substrate may avoid privileged arithmetic instructions, but it cannot avoid structure.

Therefore, a machine experiment can test whether one higher-level theory emerges without being explicitly supplied. It cannot demonstrate that algebra arises from a non-algebraic machine.

8.3 Algebra Already Present in Human Cognition and Practice

Replacing the computer with pencil and paper does not remove the problem. It moves the prior structure into the human observer and the practical environment.

A person performing the experiment already possesses capacities for:

- distinguishing marks;
- grouping;
- ordering;
- comparing;
- repeating actions;
- preserving identity;
- recognizing patterns;
- following rules;
- carrying intermediate results.

Some of these capacities may be learned. Others may reflect deeper biological organization. In either case, the observer does not approach the table as an entirely unstructured receiver.

The act of writing three marks already depends on treating multiple physical inscriptions as instances of stable symbolic types. The act of reading a table depends on spatial organization. The act of substitution depends on preserving sameness across separate occurrences.

The observer may attempt to suppress familiar concepts such as addition or multiplication. Yet the experimental practice still uses composition, sequence, equivalence, and iteration.

This creates a deeper form of inherited structure. The mathematical framework is not only in the symbols. It is also in the capacities through which those symbols become usable.

The human brain need not contain an explicit symbol + before learning arithmetic. It may nevertheless possess cognitive organization that makes combination, succession, quantity, and recurrence available.

Thus, a pencil-and-paper experiment can reduce technological assumptions but cannot produce a pre-mathematical observer.

The investigator is already situated inside a world of mathematical possibility.

8.4 NAND Gates, State Transitions, and Lower-Level Structure

A NAND-only machine offers one of the strongest attempts to avoid built-in arithmetic. NAND does not directly mean addition. It implements a simple Boolean transformation:

the output is false only when both inputs are true.

Because NAND is functionally complete, more complex logic can be constructed from it. Negation, conjunction, disjunction, exclusive OR, memory, and eventually arithmetic circuits can all be derived.

This makes NAND attractive for an operation-first thought experiment. One can ask whether an addition circuit can be constructed without addition being supplied as a primitive.

The result would be significant, but its meaning must remain precise.

The NAND substrate already contains:

- two distinguishable states;
- a binary operation;
- stable wiring;
- fan-out or signal copying;
- composition;
- temporal persistence or clocking;
- a criterion for equal outputs.

Addition could emerge as a derived circuit within that substrate, but algebra would not emerge from non-algebra. One algebraic organization would be constructed from another.

The same applies to cellular automata. A local update rule may generate complex global behavior, but the automaton already possesses states, neighborhoods, transitions, and iteration.

These systems demonstrate relative emergence:

local rule → global pattern;

primitive operation → derived operation;

lower-level transition system → higher-level algebraic organization.

They do not demonstrate absolute emergence from structurelessness.

Their value lies precisely in showing how much higher organization can be generated from a minimal operational basis.

8.5 The Regress from Theory to Substrate

Every attempt to locate the beginning of mathematics leads to a deeper layer.

A discovered law depends on a term grammar.

The term grammar depends on operations and variables.

The operations depend on a representation.

The representation depends on software or human practice.

The software depends on hardware.

The hardware depends on physical processes.

The physical processes depend on regularities describable through mathematics.

The sequence forms a regress:

theory ← language ← computation ← machine ← physics.

At no point does one reach a level entirely free of structure.

This regress does not prove that mathematics exists independently as a Platonic realm. It proves only that every accessible account of mathematical emergence begins inside prior organization.

One may continue asking whether physical law itself is mathematical, whether mathematics is imposed by observers, or whether both arise from a deeper ontology. These questions belong to metaphysics rather than to the finite algebra experiment.

The practical consequence is methodological restraint. The program should identify the level at which novelty is claimed.

For example:

- the operation was not named;
- the identity was not supplied;
- the theory was not targeted;
- the classification was not anticipated.

These are testable claims.

The claim that algebra emerged without prior mathematical structure is not.

8.6 Ontological Emergence Versus Epistemic Emergence

Ontological emergence concerns what comes into existence.

Epistemic emergence concerns what becomes known, explicit, or representable to an observer.

If an operation table already satisfies associativity, then the law is ontically true of the operation before anyone checks it. When the investigator verifies the equality and names it associativity, the law becomes epistemically explicit.

The table has not acquired a new behavior. The observer has acquired a new articulation.

This distinction is essential for interpreting Algebraic Abiogenesis.

The program may produce epistemic emergence:

- a hidden regularity becomes visible;
- an unnamed operation becomes classified;
- a large table becomes compressed into a short law;
- an implicit constant becomes definable;
- a family of models becomes recognizable as a theory.

Ontological emergence would require a stronger claim: that the algebraic structure itself came into existence through the discovery process.

In most finite-table experiments, that is false. The operation was fully specified before analysis. Its term functions and identities were already determined.

A dynamic system might generate a new operation during execution, but even then the resulting operation arises through prior transition rules. The novelty is relative to the earlier state, not absolute.

The safer formulation is therefore:

the experiment produces mathematical disclosure rather than ontological creation.

This preserves the reality of discovery while avoiding the claim that observation brings the law into being.

8.7 The Oxymoron Problem in Abiogenic Algebra

The phrase Abiogenic Algebra risks contradiction because algebraic emergence must occur through operations, relations, or transformations that are already algebraically describable.

If abiogenic means arising from what is wholly non-algebraic, then the phrase may be an oxymoron. The process would require enough structure to generate algebra while somehow containing no algebraic structure beforehand.

If abiogenic is understood relatively, the phrase becomes coherent. It can mean:

the emergence of a higher-level algebraic theory from a lower-level operational substrate in which that theory was not explicitly named or privileged.

This is analogous to biological abiogenesis. Life does not emerge from literal nothing. It emerges from chemistry, which already contains complex organization, constraints, and transformations. The term marks a transition between levels, not a transition from absolute absence to presence.

The same qualification is needed here.

Relative Algebraic Abiogenesis may describe:

- the emergence of an identity from term behavior;
- the emergence of a compact theory from an exhaustive table;
- the emergence of a derived operation from primitive operations;
- the emergence of a classification from anonymous models;
- the emergence of a stable concept from repeated operational use.

The term remains rhetorically powerful, but its technical meaning must be explicit.

Without qualification, it suggests more than the experiment can establish.

With qualification, it names a legitimate research problem:

How does higher-level algebraic organization become explicit within a lower-level structured substrate when the resulting theory has not been supplied as the target?

The answer belongs partly to mathematics, partly to computation, and partly to philosophy.

The mathematical component studies the exact passage from operations to terms, identities, clones, varieties, and classifications.

The philosophical component asks what it means for structure to emerge, to be recognized, and to become available as mathematics.

The phrase is therefore not useless, but it must no longer carry the illusion of an absolute beginning.

9. From Algebraic Birth to Mathematical Disclosure

The language of birth suggests that an algebraic object, operation, or theory did not exist and then came into existence through a process of discovery. For operation tables, this description is usually too strong. Once the carrier and table are fixed, every primitive output, derived term function, valid identity, and definable operation is already determined. The investigator does not cause these structures to become true by calculating them.

What changes is their mode of availability. A regularity that was distributed across many table entries becomes concentrated in a single equation. A term that previously functioned only as a calculation becomes recognizable as a constant or derived operation. A collection of individual cases becomes organized as a theory. Structure that was implicit in use becomes explicit in language.

Mathematical disclosure names this transition more accurately than algebraic birth. Disclosure does not imply that the structure was wholly absent before it was recognized. It means that something previously operative but unarticulated becomes available as a stable object of thought, communication, proof, and further construction.

The shift is from extensional dispersion to intensional organization:

table entries → recurring behavior → explicit law → mathematical object → theory.

This framework preserves the reality of discovery while rejecting the claim that algebra has emerged from non-algebra. The discovery lies in articulation, compression, stabilization, and recognition.

9.1 Implicit Structure and Explicit Articulation

A finite table may contain a law without displaying it directly. Associativity, for example, is not written in any single cell. It concerns the relation between two compound procedures:

$(x \diamond y) \diamond z$

and

$x \diamond (y \diamond z)$.

If the two expressions agree for every assignment, associativity is implicit in the complete behavior of the table. The investigator makes it explicit by stating one general equation.

Implicit does not mean vague, incomplete, or merely possible. The law is already determined by the operation. Its truth does not depend on whether anyone has noticed it.

What is absent is articulation. Before the equation is written, the regularity is distributed across many individual evaluations. It has not been isolated as one reusable proposition.

Articulation performs several functions. It compresses cases, permits proof, supports comparison with other structures, and allows the regularity to be communicated independently of the original table. The law becomes transportable.

The same distinction applies to physical addition. A conserved quantity may behave additively before anyone states the corresponding equation. The formal law does not create the regularity. It articulates a stable relation within a conceptual system.

Mathematical discovery often consists in finding the expression under which dispersed behavior becomes intelligible as one structure.

9.2 Operational Regularity Before Theoretical Naming

An operation can display regular behavior before the investigator possesses a name for that behavior.

Repeated calculation may reveal that changing parentheses never changes the output. The investigator may initially record only:

E_1 and E_2 always agree.

Later, the pattern is expressed as an identity. Still later, it is recognized as associativity.

These stages should be distinguished:

1. the operation behaves regularly;
2. the regularity is observed;
3. the regularity is expressed symbolically;
4. the law is connected to an established concept;
5. the operation is classified within a known theory.

The first stage belongs to the operation. The later stages belong to the process of mathematical understanding.

Naming is therefore not the beginning of structure. It is a moment of integration. By calling a law associative, the investigator places it into a historical network of theorems, examples, counterexamples, and classifications.

Before that naming, the regularity is not meaningless. It already appears through repeated successful use. The name makes the pattern theoretically legible.

This is why delayed classification can be useful. It allows operational behavior to be recorded without immediately forcing it into a familiar category. But the delay does not imply that the operation exists outside all interpretation. It remains situated within the practice of evaluation and comparison.

9.3 Disclosure as the Stabilization of a Mathematical Object

A mathematical object becomes stable when it can be identified across different presentations, calculations, and contexts.

Consider the term:

x ⋄ x.

Suppose every substitution produces the same carrier element. Initially, each evaluation is a separate event:

a ⋄ a gives one mark;

b ⋄ b gives the same mark;

c ⋄ c gives the same mark.

When the investigator proves that the result is independent of x, the term acquires a stable role. It can be treated as a definable constant:

c := x ⋄ x.

The symbol c does not create the element. It stabilizes a pattern already present in the operation.

Stabilization means that the object can now be used without repeating the entire discovery procedure. One may refer to c, prove further laws involving c, compare it with identities or absorbers, and transport it into another representation.

A theory performs the same operation at a larger scale. It stabilizes a collection of laws, models, and transformations as one coherent object of study.

Disclosure therefore has a constructive dimension. Although the underlying behavior is already determined, the mathematical object as an explicit, reusable unit of reasoning is formed through articulation.

This is not creation from nothing. It is object formation through invariant recognition.

9.4 The Emergence of Constants, Identities, and Derived Operations

Operation-first experiments can reveal structures that were not primitive in the original language.

A constant may emerge when a variable term has the same value under every assignment. If:

$x \diamond x = y \diamond y$

for all x and y, then the common value can be named c.

An identity element may emerge when that constant or another definable element satisfies:

$c \diamond x = x$

or:

$x \diamond c = x$.

A unary operation may emerge by fixing one argument:

$n(x) := x \diamond c$.

A second binary operation may emerge from a compound term:

$x \bigstar y := (x \diamond y) \diamond x$.

These objects are derived rather than primitive. They were not listed separately in the initial signature, but they were constructible from the supplied operation.

The term emergence is appropriate if used relatively. The constant, unary map, or secondary operation becomes available at a higher descriptive level even though its values were already determined by the original table.

The important question is whether the derived object introduces explanatory power. A useful derived operation may simplify the law system, expose symmetry, reveal a familiar representation, or support a more compact axiom basis.

Not every definable term deserves promotion to an object. A finite algebra may contain many derived operations. Mathematical disclosure requires selection. The investigator stabilizes those terms that organize further reasoning.

Thus emergence involves both definability and significance.

9.5 Recognition Versus Invention

Recognition identifies a structure already realized in the supplied behavior. Invention introduces a new representation, definition, question, or construction through which that structure becomes accessible.

The distinction is not absolute. Mathematical work often combines both.

If an anonymous table is shown to be subtraction modulo three, the investigator has recognized a familiar structure. The table was already isomorphic to that operation.

If the investigator introduces an unexpected derived operation that converts the table into a simpler form, this may involve invention. The new operation was definable from the original one, but the choice to privilege it as explanatory was not forced by the table alone.

The table determines what is possible. It does not uniquely determine which description should be considered natural.

Several mathematically equivalent presentations may exist:

- a complete operation table;
- a numerical formula;
- a short identity basis;
- a derived group operation;
- a permutation representation;
- a clone-theoretic classification.

Choosing among these requires judgment about simplicity, explanatory power, and future usefulness.

Mathematical discovery is therefore neither pure recognition nor unrestricted invention. It is constrained invention directed toward the recognition of stable structure.

The operation resists arbitrary interpretation because incorrect descriptions fail. Yet the operation does not dictate one inevitable language in which it must be understood.

9.6 Rediscovery Under Unfamiliar Notation

Anonymous notation creates a controlled risk of rediscovery. By suppressing familiar symbols and names, the investigator may reproduce known mathematics without recognizing it.

This is useful during the exploratory stage. If a system recovers associativity, a group-like law, or a known clone without being told to search for it, the result demonstrates that the method can detect genuine structure.

But rediscovery is not historical novelty.

A table may look unfamiliar because its carrier elements have been renamed. Its inputs may be reversed. Its outputs may be recoded. A familiar operation may appear only after introducing a derived constant or unary map.

For this reason, every candidate discovery requires a recognition audit. One should test:

- simultaneous relabeling of carrier elements;
- reversal of input order;
- independent recoding of inputs and outputs;
- definable constants;
- short derived unary and binary operations;
- equivalence of generated clones;
- mutual implication of candidate law systems.

The purpose is not to discredit rediscovery. It is to identify what kind of accomplishment has occurred.

Autonomous rediscovery can validate a method. A shorter presentation can be new even when the underlying theory is known. A new finite census can be original even when every classified object is classical.

The claim must match the result.

Unfamiliar notation suspends recognition. It does not suspend mathematical history.

9.7 Mathematical Disclosure as a Replacement Framework

Mathematical disclosure provides a more precise framework than absolute Algebraic Abiogenesis.

Its basic sequence is:

operative structure → systematic engagement → noticed regularity → symbolic articulation → stabilized object → historical interpretation.

Each stage can be studied without claiming that mathematics arose from nothing.

The framework distinguishes several legitimate achievements:

- disclosure of an identity not supplied as a target;
- disclosure of a derived operation hidden in a primitive presentation;
- disclosure of a compact law basis within an exhaustive table;
- disclosure of a known theory under unfamiliar representation;
- disclosure of a previously unclassified structure;
- disclosure of how background operations determine what appears fundamental.

This approach also clarifies the division between mathematics and philosophy.

The mathematical questions concern exact operations, term functions, identities, definability, clones, varieties, countermodels, and classifications.

The philosophical questions concern what it means for those structures to become visible, whether recognition differs from creation, and how practical engagement precedes formal objectification.

The two inquiries can inform one another without being confused.

Mathematical disclosure does not weaken the research program by replacing a dramatic origin metaphor with a modest epistemic description. It identifies more accurately where the substantive work occurs.

The operation was present. Its laws were true. But neither the operation's structural role nor the law's explanatory significance was necessarily available to the investigator.

The event of discovery consists in making that organization explicit and stable enough to enter mathematics as a shared object.

The revised central claim is therefore:

Algebra is not born when an anonymous table is examined. An already operative structure becomes algebraically disclosed when its regularities are articulated, stabilized, and recognized within a formal and historical world.

10. Human and Artificial Mathematical Discovery

Artificial mathematical discovery is often described as though the machine enters a domain of neutral symbols and independently finds structure. This description is only partly accurate. A

computer may generate expressions without human intuition, test millions of cases, identify term collisions, search for countermodels, and rank candidate theories. Yet the machine's field of inquiry is still defined by a human-designed language, a representation of objects, an evaluation procedure, and a criterion for significance.

The computer is therefore neither a passive calculator nor an autonomous mathematical consciousness. It is a structured observer operating within a supplied world.

This position avoids two extremes. The first treats the machine as merely executing human mathematics and therefore incapable of discovery. The second treats it as creating mathematics from an interpretation-free void. In practice, artificial discovery occupies an intermediate position. The machine can reveal consequences, regularities, and organizations that were not individually anticipated by its designers, while remaining dependent on the conditions under which those patterns can appear.

The important question is not whether a machine is completely independent of human structure. No mathematical investigator is. The more useful question is whether the machine can produce results that were not explicitly encoded as targets, recognize new regularities, reorganize its representational resources, and generate mathematically consequential objects that survive external verification and historical comparison.

10.1 The Computer as a Structured Observer

A computer does not encounter an operation table as raw reality. The table is represented in memory according to a chosen encoding. Carrier elements become integers, strings, bit patterns, or indexed objects. Rows and columns become data structures. Equality becomes a machine procedure. Terms become trees, graphs, strings, or compiled functions.

The machine therefore observes through a technical apparatus.

Its basic acts may include:

- reading an encoded symbol;
- selecting an array entry;
- evaluating a term;
- comparing signatures;
- storing collisions;
- rejecting candidates;
- ranking outputs.

Each act presupposes distinctions supplied by the representation.

This makes the computer comparable to a scientific instrument. A microscope does not display reality without mediation. It reveals what can appear through its optics, preparation methods, contrast mechanisms, and resolution. Likewise, a theorem-discovery program reveals what can appear through its grammar, search depth, normalization rules, equivalence tests, and computational limits.

The machine may be objective in the sense that it applies these procedures consistently. It is not neutral in the sense of being free from a prior mode of access.

Its observations are exact relative to the representation and policy under which it operates.

10.2 Human Projects Embedded in Computational Searches

Every computational search begins inside a project.

A human chooses to investigate finite binary operations rather than topological spaces, stochastic processes, or differential equations. A human chooses whether to search for identities, inequalities, invariants, generators, countermodels, or efficient circuits. A human decides which outputs are interesting enough to preserve.

Even when the machine generates candidate expressions autonomously, the generation process answers a human-framed question:

What regularities become visible within this formal world?

The project may be broad, but it is never absent.

For Algebraic Abiogenesis, the guiding project has included aims such as:

- begin from anonymous finite operations;
- generate expressions without selecting a target theorem;
- identify exact term-function collisions;
- extract candidate laws;
- reject accidental patterns through countermodels;
- compare emergent theories;
- discover whether higher algebraic organization can form without prior naming.

These aims give the machine's activity its meaning. Without them, the same computations would be only state transitions.

The machine does not need to hold the project consciously. The project is embedded in the design of the experiment.

10.3 Why Anonymous Operations Remain Policy-Dependent

An anonymous operation is never examined from every possible perspective at once. It is observed through a policy.

A policy determines what counts as visible, equivalent, simple, admissible, or important.

For example, a search policy may specify:

- which variables are allowed;
- which term depths are explored;
- whether constants are included;
- whether repeated variables are permitted;
- whether equivalent syntax trees are merged;
- whether operation cost is measured by depth, size, or evaluation time;
- which term collisions are retained;
- which models are used for counterexample testing.

Changing the policy can change the apparent theory.

A law invisible at depth three may become obvious at depth five. A constant may remain hidden when nullary symbols are forbidden but become definable through a unary term. Two operations may appear distinct under primitive-table comparison but identical under clone equivalence.

Thus, the discovered mathematics is constrained by the operation and conditioned by the policy.

The operation does not become arbitrary. A false identity will still fail when tested exhaustively. But the set of identities that becomes visible depends on how the search is organized.

This policy dependence should be documented rather than concealed.

10.4 Background Signatures and Hidden Expressive Resources

The expressive power of a primitive operation cannot be assessed independently of the background operations supplied with it.

Suppose a system studies pairs of binary operations while providing negation freely. A pair may appear noncooperative because each member can define the desired target operations with the help of negation alone.

If negation is removed, the same pair may become irreducibly cooperative. Neither operation suffices individually, but together they generate a larger expressive system.

The apparent fundamentality of an operation is therefore background-relative.

Let B denote the background signature, G the candidate generating operations, and P the observational policy. The resulting theory or expressive closure may be regarded schematically as:

T(B, G, P).

Changing B can alter which generators appear essential. Changing P can alter which consequences are detected. Changing G can alter the generated operational world.

This is not merely a software concern. It is a mathematical fact about definability and generation.

A study claiming that an operation is primitive, redundant, cooperative, or expressive must therefore identify the background against which that judgment is made.

Background ablation becomes a central method. One removes supposedly free resources and asks which structures survive, disappear, or become newly visible.

10.5 Artificial Intelligence as an Instrument of Articulation

Artificial intelligence can extend mathematical disclosure by operating across scales inaccessible to unaided human attention.

A human may inspect a small table and notice one or two patterns. A machine can generate thousands of terms, evaluate them over every assignment, cluster equal term functions, construct candidate bases, and search for separating models.

This capacity makes the machine an instrument of articulation.

The operation table already determines its term behavior, but that behavior may be too large for direct comprehension. The machine converts the implicit structure into explicit records:

- equivalence classes of terms;
- minimal representatives;
- candidate identities;

- dependency graphs;
- counterexamples;
- generated subtheories;
- signature refactorings.

The machine does not create the truth of these outputs. It makes them available at a new scale and level of organization.

This role resembles the use of telescopes, particle detectors, or sequencing machines. The instrument does not invent the observed pattern, but it constructs a channel through which the pattern becomes visible and reproducible.

The strongest artificial-discovery systems may go further by proposing new representations, changing search languages, selecting intermediate concepts, or revising what counts as a productive question.

At that point, the machine participates not only in calculation but in the formation of the mathematical field of visibility.

10.6 Human Judgment and Historical Recognition

A machine can determine that two terms agree on every assignment in a finite algebra. It can certify that a candidate identity follows from an axiom set or find a countermodel showing that it does not.

It may be much less reliable at deciding whether the result is historically new, conceptually important, or mathematically illuminating.

These judgments require broader comparison.

A newly generated law may be:

- a standard identity under unfamiliar notation;
- an immediate consequence of known axioms;
- a special case of a classical theorem;
- a redundant member of an already classified theory;
- a genuinely new finite phenomenon;
- a new presentation of a known structure;
- a useful computational result without deep theoretical novelty.

Historical recognition places the result within an inherited mathematical world.

Human expertise remains important because significance is not identical to truth. Infinitely many true equations may hold in a finite algebra. Only some reveal structure worth preserving.

The machine can assist by measuring simplicity, independence, explanatory coverage, or frequency across models. Yet the final judgment may still depend on conceptual relationships that are not captured by those metrics.

The discovery process therefore requires both exact machine verification and disciplined historical interpretation.

10.7 What Machines Can Disclose Without Understanding

A machine may disclose a law without understanding it in the human sense.

It can generate the terms, calculate their signatures, identify equality, minimize the expression, and produce a proof certificate. None of these steps requires the machine to experience surprise, grasp significance, or situate the law within a lived mathematical practice.

This raises a distinction between functional disclosure and conscious understanding.

Functional disclosure occurs when a system reliably transforms hidden regularity into explicit, verifiable form.

Understanding may additionally involve:

- recognizing why the law matters;
- seeing analogies with other theories;
- anticipating productive generalizations;
- explaining the result through an illuminating concept;
- revising one's broader mathematical worldview.

A machine can contribute substantially at the first level even if its status at the second remains uncertain.

The absence of human-like understanding does not make the output mathematically unreal. A proof certificate remains checkable. A countermodel remains valid. A newly classified algebra remains an object of study.

Yet the distinction matters when describing agency. The machine may be the causal source of a discovery event without possessing the same mode of comprehension as a mathematician.

It can disclose structure operationally before anyone understands that structure conceptually.

10.8 The Separation of Mathematical Result and Philosophical Interpretation

The mathematical result and its philosophical interpretation must be evaluated separately.

A program may establish that:

- a finite census is complete;
- one algebra uniquely maximizes a specified score;
- a candidate identity is independent of others;
- a finite separator exists within a proven bound;
- removing a background operation changes generator cooperation;
- a newly defined invariant distinguishes previously grouped cases.

These are mathematical or computational claims. They can be checked through proof, exhaustive enumeration, replication, or formal certification.

A philosophical interpretation may then claim that the result demonstrates emergence, disclosure, artificial creativity, or the formation of mathematical objects.

That interpretation is not proved merely by the computation.

Conversely, philosophical criticism of the word abiogenesis does not invalidate a correct finite census or exact theorem. A weak metaphor does not make the mathematics false.

The research program therefore needs a strict separation:

mathematical layer → operations, algorithms, identities, invariants, proofs, classifications;

philosophical layer → emergence, meaning, recognition, agency, disclosure, ontology.

The layers may interact, but neither should substitute for the other.

A philosophical narrative cannot rescue a computational result that lacks novelty. A mathematically novel result does not automatically settle philosophical questions about machine understanding or the origin of structure.

The most defensible account of human and artificial discovery is cooperative but asymmetrical.

The human supplies much of the project, language, background, and historical judgment. The machine supplies scale, exactness, exhaustive comparison, and the ability to reveal patterns that no individual investigator explicitly anticipated.

Neither begins from nothing.

Both operate within inherited structures. Both may nevertheless encounter consequences that were not previously available to them.

Artificial mathematical discovery is therefore not the birth of mathematics inside a machine. It is a new site at which mathematically determined structures can become explicit, testable, and available for recognition.

11. A Discipline for Operation-First Experiments

Operation-first inquiry requires stronger controls than ordinary theorem proving because the method begins by suppressing familiar interpretation. That suppression is useful for reducing bias, but it also creates a risk: inherited structure may disappear from attention and later be reported as though it had emerged spontaneously.

A disciplined experiment must therefore preserve a complete record of what was supplied, what was generated, what was observed, and what was inferred. It must distinguish a primitive operation from its derived operations, a true identity from an interesting identity, rediscovery from novelty, and mathematical proof from philosophical interpretation.

The central requirement is traceability. Every reported result should be traceable back through the expression grammar, evaluation policy, background signature, carrier, and original operation tables that produced it. If the experiment includes ranking, compression, or theory selection, those procedures must also be explicit.

The purpose of this discipline is not to eliminate interpretation. That would be impossible. Its purpose is to expose the interpretive and formal conditions under which a mathematical structure became visible.

A rigorous operation-first study should answer five questions:

What was present before the search?

What was permitted to vary?

What counted as an observation?

What transformation produced the reported object?

What evidence distinguishes novelty from disguised familiarity?

Without these distinctions, anonymous notation can become a machine for manufacturing exaggerated claims. With them, operation-first experimentation can become a reproducible method for discovering, classifying, and comparing finite algebraic structure.

11.1 Recording Everything Supplied Before Discovery

The first stage is a complete inventory of prior structure. This inventory should be written before the search results are inspected.

At the mathematical level, the record should specify the carrier, signature, arities, totality or partiality of each operation, permitted constants, equality policy, and rules of term formation. If operations are presented by tables, the table orientation must be fixed: whether rows represent first inputs and columns represent second inputs cannot be left implicit.

The record should also state whether outputs belong to the original carrier. If closure is guaranteed by the data format, closure cannot later be reported as an emergent property.

At the computational level, the record should identify the encoding of elements, normalization procedures, expression-generation order, maximum term depth or size, treatment of repeated variables, and stopping conditions.

At the conceptual level, the study should state its project. A search for shortest identity bases is not the same experiment as a search for expressive completeness. A search for generator cooperation is not the same as a search for new varieties. The same operation may disclose different structures under different investigative aims.

The prior-structure record should also include resources that are easy to overlook. These include free background operations, built-in constants, equality testing, variable projections, argument permutation, and any canonicalization used to merge expressions.

Suppose negation is supplied without cost. A binary operation may then appear sufficient to generate a target theory only because negation is silently doing essential work. Unless that background is recorded, the conclusion about primitiveness will be misleading.

The discipline is simple:

nothing counted as available during the experiment should be omitted from the initial ledger.

This does not make discovery impossible. It makes the location of discovery visible.

11.2 Separating Computation from Interpretation

The experiment should separate raw computational output from later mathematical interpretation.

The computational layer may report that two terms have identical signatures, that one finite algebra maximizes a score, or that no countermodel exists below a stated size. These are direct products of the procedure.

The interpretive layer may describe the collision as associativity, classify the algebra as a known quasigroup, or argue that the result demonstrates background-relative emergence.

These statements are not equivalent.

A recommended record contains at least three levels.

The first level is observation:

the terms s and t evaluate identically under all assignments in the tested algebra.

The second level is mathematical inference:

the algebra satisfies the identity s = t.

The third level is interpretation:

the identity is an instance of associativity, belongs to a known theory, or supports a broader account of disclosure.

Keeping these levels separate prevents historical or philosophical language from entering the computational evidence.

The same separation should apply to negative results. A bounded search that finds no counterexample has not necessarily proved a universal theorem. It has proved only what the search design certifies. If a complete finite enumeration has been performed, completeness must itself be justified.

Interpretation remains essential. A machine-generated list of equalities is not yet an intelligible theory. But interpretation should begin after the exact output has been preserved, not replace that output.

11.3 Neutral Expression Generation

Neutral expression generation aims to avoid favoring identities already familiar to the investigator.

For a language containing a single binary operation $\diamond$, expressions can be generated recursively from variables. At the first level are the variables themselves. At the next level are expressions such as $x \diamond y$. At higher levels, previously generated expressions are inserted into either argument position.

Neutrality requires a declared generation rule. The system should not preferentially generate associative shapes, commutative comparisons, or terms resembling known axioms unless that bias is part of the stated experiment.

Several measures of complexity are possible:

term depth;

number of operation symbols;

number of distinct variables;

total symbol count;

evaluation cost;

description length.

The chosen measure affects which expressions appear early and which identities are judged simple.

Duplicate syntax should be handled carefully. Two expression trees may differ only by variable renaming. Depending on the aim, they may be retained separately or placed in a common orbit under renaming. The choice changes the search space.

Neutral generation also requires caution about constants. If no constants are primitive, the generator should not silently insert carrier elements into terms. Constants may emerge only if a variable term is provably independent of its assignments or if the experimental language explicitly permits named elements.

Expression generation defines the horizon of possible discovery. A law requiring deeper nesting cannot appear in a shallow search. A ternary phenomenon cannot be detected if only two variables are permitted. A relation cannot appear if the language contains only operations and equations.

The search grammar should therefore be treated as part of the mathematical object under study, not as an invisible programming convenience.

11.4 Collision Registers and Law Extraction

Each generated term induces a function on the carrier. For a finite algebra, that function can be represented by its complete output signature over all variable assignments.

Terms with identical signatures belong to the same semantic class.

A collision register should record:

the terms compared;

their complexity;

their variable sets;

their complete signatures or a cryptographic digest linked to stored signatures;

the first stage at which their equality was detected;

the models in which the equality was tested;

whether the identity was later proved from a proposed basis.

The collision register is more than a list of equations. It preserves the relation between syntax and semantic collapse.

When many terms fall into the same class, the system may choose one representative. The representative may be the shortest term, the shallowest, the lexicographically earliest, or the one judged most interpretable. That choice must be recorded because it shapes the resulting theory.

Law extraction should avoid reporting every collision. A finite algebra satisfies infinitely many redundant identities. The meaningful problem is to identify a compact collection from which substantial portions of the remaining equational behavior follow.

Candidate laws may be ranked by simplicity, independence, coverage, stability across related models, or ability to compress the collision register.

A law should not be called fundamental merely because it appeared early. Early appearance may reflect the term-generation order rather than structural importance.

The register must preserve enough information to revisit that judgment.

11.5 Relabeling, Opposite Operations, and Derived-Operation Audits

A candidate operation should be tested under transformations that preserve or reveal hidden equivalence.

The simplest transformation is carrier relabeling. If the elements a, b, and c are renamed, the resulting table may look different while defining an isomorphic algebra.

A second transformation reverses argument order. From $\diamond$ one forms the opposite operation:

$x \diamond^{op} y = y \diamond x$.

A noncommutative operation and its opposite may belong to the same broad theory or may become isomorphic under relabeling.

A third transformation changes the primitive presentation through derived operations. A complicated-looking operation may become familiar once a definable constant, inverse-like map, or alternative binary operation is introduced.

For example, one may define:

$c := x \diamond x,$

$n(x) := x \diamond c,$

$x \bigstar y := n(x) \diamond y.$

The resulting operation ★ may reveal a group, lattice, Boolean, or quasigroup structure obscured in the original presentation.

The audit should test short derived terms systematically rather than relying only on human ingenuity. It should ask whether the candidate algebra becomes a known structure under a low-complexity change of signature.

Independent recoding of row labels, column labels, and output labels may also be useful when studying quasigroups and related combinatorial structures. Such transformations can reveal isotopy even when strict isomorphism is absent.

A candidate that survives these audits is not automatically new. But a candidate that fails them should not be presented as a newly discovered algebra merely because its original table looked unfamiliar.

11.6 Testing for Constants, Projections, Identities, and Familiar Structures

Before advanced interpretation begins, every operation should undergo a basic structural examination.

The system should test whether the operation is constant, a left or right projection, commutative, associative, idempotent, medial, cancellative, or absorptive. It should search for left and right identities, zero-like elements, inverses, fixed points, and definable constants.

These tests are not intended to force the operation into a predetermined class. They provide a minimal recognition profile.

The search should also inspect the rows and columns as transformations. Repeated rows may indicate loss of information. Permutation rows may indicate quasigroup or group-like behavior. Constant rows may reveal absorbers. Symmetries may indicate automorphisms.

Derived unary operations should be tested for involution, idempotence, cycles, and fixed points. Derived binary operations should be examined for familiar laws.

A simple operation may conceal a richer structure, while an apparently complex operation may collapse into a projection or constant after one substitution.

The audit should proceed from elementary to more specialized recognition. Early elimination of trivial and familiar cases reduces the risk of building philosophical claims around structures that are mathematically routine.

11.7 Distinguishing Syntactic Novelty from Semantic Novelty

A new expression is not necessarily a new operation. A new table is not necessarily a new algebra. A new axiom list is not necessarily a new theory.

Syntactic novelty concerns presentation. It may involve a new symbol, term, formula, or arrangement of axioms.

Semantic novelty concerns the structure or class of models determined by that presentation.

Two different terms may define the same function. Two different tables may be isomorphic. Two different identity bases may generate the same equational theory. Two different primitive signatures may generate the same clone.

The experiment should therefore specify the level at which novelty is claimed.

Possible levels include:

new shortest term for a known operation;

new primitive basis for a known clone;

new finite algebra up to isomorphism;

new identity independent of a stated basis;

new finite-model census;

new invariant;

new equational theory;

new variety or quasivariety;

new algorithmic method.

These levels differ greatly in importance, but each can be legitimate when stated accurately.

The discipline of novelty requires asking not merely whether the output has been seen in the current run, but whether its semantic content has already appeared under another representation.

11.8 Historical and Literature Comparison

No mathematical discovery claim is complete until the result is compared with prior work.

The comparison should begin with structural descriptors rather than only the newly assigned name. Searching for the operation's invented name will reveal nothing if the structure was previously studied under a different terminology.

The audit should use:

identity patterns;

carrier size;

known varieties;

clone classifications;

quasigroup and loop terminology;

semigroup classifications;

logical algebra terminology;

finite model databases;

standard transformation equivalences.

The literature search should also examine adjacent fields. A structure presented as an algebra may already be known as a logic, circuit basis, combinatorial design, automaton, or constraint language.

Failure to find a match does not prove novelty. It indicates only that the search has not yet located a predecessor.

When the literature is too large or terminology too unstable for certainty, the claim should remain provisional.

Rediscovery can still be valuable. If a machine independently recovers a known structure without being targeted toward it, that may validate the discovery process. The publication claim should then concern autonomous recovery, efficiency, representation, or methodology rather than the historical invention of the structure.

11.9 Standards for Claiming Genuine Mathematical Discovery

A strong claim of mathematical discovery should satisfy several standards.

The result must be exact. A proposed identity should be proved, exhaustively verified in a finite model, or accompanied by a certificate appropriate to the claim.

The result must be reproducible. Another investigator should be able to reconstruct the carrier, operations, grammar, search policy, and outputs.

The result must survive equivalence audits. Relabeling, opposite operations, derived signatures, and known classifications should be tested.

The result must be nontrivial relative to the background. A structure should not be called emergent if it was effectively supplied through free operations, constants, or scoring rules.

The result must be historically compared. Novelty requires more than local unfamiliarity.

The result must be stated at the correct level. A new representation should not be called a new algebraic theory. A complete census should not be called a new variety unless a new variety has actually been established.

The result should also have mathematical consequence. It should prove something not previously available, classify a meaningful family, introduce a useful invariant, sharpen a bound, supply a new algorithm, or reveal a structural relationship that supports further work.

Finally, philosophical interpretation should remain proportional to the mathematics.

A genuine theorem may support a philosophical discussion of disclosure. It does not prove that mathematics arose from nothing, that a machine understands the result, or that a new ontology has been created.

The highest standard for operation-first work is therefore not dramatic language. It is transparent experimental design followed by exact mathematics and restrained interpretation.

Under this discipline, anonymous operations become useful instruments of inquiry. They suppress familiar labels without pretending to eliminate inherited structure. They allow laws to be detected before classification while preserving the audit trail needed to distinguish recovery, reformulation, and genuine novelty.

12. Toward a Phenomenology of Mathematical Formation

A phenomenology of mathematical formation asks how mathematical objects, laws, and theories become available to inquiry before questions of proof, classification, and historical

novelty are settled. It does not replace mathematics with subjective experience. Nor does it claim that mathematical truth depends on a particular observer's feelings. Its concern is the mode of appearance through which a structure becomes identifiable as a mathematical object.

Operation-first experiments provide a particularly clear setting for this inquiry. An anonymous table is already usable before it is understood. Terms can be evaluated before their identities are known. Repeated outputs can be noticed before they are compressed into laws. A familiar algebra can be enacted before it is recognized under its historical name.

The important transition is therefore not from nothing to mathematics. It is from unthematized operational involvement to explicit mathematical articulation.

This transition has several stages. Distinctions are stabilized. Procedures become repeatable. Regularities become conspicuous. Expressions are identified across cases. Laws are stated. Objects are named. Structures are classified. Each stage changes what is available for further reasoning.

A phenomenology of mathematical formation studies this layered process while preserving the difference between appearance and truth. An identity becomes visible through a practice, but its validity is determined by the operation. A theorem may be disclosed historically at one moment while having been mathematically true beforehand.

The result is an account of mathematics as structured disclosure: objective constraint becoming explicit through situated practices of symbol use, comparison, proof, and recognition.

12.1 Mathematics as Articulation Rather Than Creation from Nothing

Mathematical language often uses metaphors of creation. Mathematicians construct objects, invent definitions, create theories, and discover new worlds. These expressions capture genuine intellectual activity, but they can obscure the difference between introducing a representation and bringing a structure into existence.

When a finite operation table is fixed, its behavior is already determined. Every possible evaluation of every term follows from the primitive table and the rules of term formation. A later identity does not alter that behavior.

The investigator contributes articulation. A large collection of individual evaluations becomes one general statement. A complicated table becomes a compact formula. A term class becomes a named derived operation. A scattered family of examples becomes a mathematical category.

Articulation is not trivial transcription. It reorganizes what can be known and done. Once a law has been stated, it can be proved from other laws, transported into new settings, used in deductions, and compared with known theories.

The formation of explicit mathematics therefore has constructive force without requiring creation from nothing.

The investigator does not create the truth of the law. The investigator creates or selects a form in which that truth becomes available.

This distinction allows discovery and invention to coexist. The structure constrains what can be said correctly. The mathematician invents the language, representation, and conceptual organization through which the structure is disclosed.

12.2 The Observer as Part of the Mathematical Event

The observer is not external to the event of discovery in the sense of being irrelevant to what becomes visible. The operation's truth does not depend on the observer, but its disclosure occurs through an observational arrangement.

The investigator chooses a carrier representation, a signature, a term grammar, a depth bound, an equality criterion, and a method of comparison. These choices establish a field within which some structures become visible and others remain hidden.

An associative law cannot be disclosed if compound terms are never formed. A definable constant may remain unnoticed if repeated-variable terms are excluded. A clone-theoretic equivalence may remain invisible if only primitive tables are compared.

The observer therefore participates by shaping access.

This does not imply that the observer may choose any conclusion. The operation resists false descriptions. A candidate identity either survives complete evaluation or fails under a counterexample.

The mathematical event involves both situated access and objective constraint.

The observer supplies the question, language, and procedure. The structure supplies the permissible answers.

This interaction is especially clear in machine-assisted mathematics. The human may not inspect every calculation, but the observational policy remains embodied in software. The machine extends the investigator's reach while inheriting the structure of the inquiry.

12.3 Practical Intelligibility Before Formal Ontology

Formal ontology asks what kinds of mathematical objects exist within a theory: elements, operations, relations, morphisms, equivalence classes, spaces, or categories.

Yet these objects often become available through practical activity before they are explicitly declared.

An operation table is first used. A diagram is traversed. A transformation is applied. A proof technique is repeated. A computational routine is trusted through successful execution.

Only later may the investigator isolate the object that made the activity possible.

In the table experiment, $\diamond$ is practically intelligible as something used to obtain an output from two inputs. This understanding precedes a complete theoretical account of $\diamond$ as a quasigroup operation, a group-derived subtraction, or a generator of a particular clone.

Formal ontology arises when the operation is detached from the immediate act of use and described through explicit properties.

The practical and formal modes should not be treated as competitors. Practical involvement supplies access. Formalization supplies stability, precision, and transportability.

Mathematical formation often moves through both:

competent use → noticed regularity → explicit definition → formal object.

The final theory may conceal this history. Once the object is defined axiomatically, it appears self-contained. A phenomenological account restores the path by which it became visible.

12.4 Mathematical Objects as Stabilized Structures of Disclosure

A mathematical object is stabilized when it can be identified across varying instances, representations, or procedures.

Suppose the term $x \diamond x$ always yields the same element. Individual evaluations vary in their input inscriptions, but the output remains invariant. The investigator may then treat the repeated result as one constant.

The object is stabilized through invariance.

Similarly, an algebraic structure may appear under many relabelings. Its operation table changes visually, but its isomorphism class remains fixed. Classification stabilizes the structure beyond a particular presentation.

A theorem stabilizes a relation by showing that it holds under stated conditions rather than only in observed cases. A proof removes dependence on the accidents of one calculation.

Stabilization is therefore a central act of mathematical formation.

It turns:

repeated event into invariant;

expression into operation;

table into model;

law list into theory;

presentation into structure.

The object is not merely the physical mark or stored data. It is the stable organization preserved across admissible variation.

This view connects mathematical objecthood with the broader study of identity policies. What counts as the same object depends on which differences are ignored and which are preserved.

Two tables may be different as labeled arrays but the same up to isomorphism. Two terms may be different syntactically but the same as term functions. Two axiom systems may be different as lists but equivalent in deductive strength.

Mathematical formation therefore involves not only producing objects but establishing the appropriate criterion under which they remain the same.

12.5 The Role of Breakdown, Anomaly, and Counterexample

Mathematical structure often becomes visible when an expectation fails.

An operation may appear commutative until one ordered pair produces a different result under reversal. A proposed axiom basis may appear sufficient until a finite countermodel satisfies the axioms but violates the intended law. A classification may appear complete until an anomalous case resists every known representation.

Such failures are not merely obstacles. They disclose hidden assumptions.

A counterexample identifies a distinction that the candidate theory failed to preserve. It reveals that two concepts previously treated as equivalent must be separated.

Breakdown also exposes the equipment of inquiry. A software error may reveal dependence on a normalization rule. A failed replication may reveal an undocumented background operation. A surprising result may reveal that the scoring policy favored a structure for an unintended reason.

The anomaly redirects attention from ordinary use to the conditions that made that use possible.

In mathematical discovery, negative evidence can therefore be more formative than confirmation. A successful identity fits within the existing horizon. A counterexample changes the horizon.

Theories grow not only by accumulating laws but by learning which distinctions cannot be collapsed.

A mature phenomenology of mathematical formation must therefore include failure as a primary mode of disclosure.

12.6 Artificial Systems as New Sites of Mathematical Appearance

Artificial systems create new environments in which mathematical structures can become explicit.

A machine can enumerate terms beyond human patience, compare complete semantic signatures, search enormous finite spaces, and preserve every collision. It can disclose patterns that were always determined by the underlying operations but were not practically accessible to human inspection.

The machine is therefore not merely faster paper. It changes the scale and organization of appearance.

Patterns may become visible statistically across millions of models. Entire landscapes of definability may be computed. Background operations may be ablated systematically. Candidate theories may be evolved, compared, and rejected through automated countermodel search.

These capacities can produce new mathematical objects of attention:

- distributions of theories across model spaces;
- exact signature landscapes;
- minimal generator profiles;
- automated dependency graphs;
- finite separator spectra;
- structural phase transitions under policy changes.

Such objects may not have been conceptually anticipated before computation made them visible.

The machine remains embedded in a designed environment, but that does not reduce its outputs to trivial execution. Human-designed instruments have repeatedly revealed phenomena their designers did not predict.

The question is not whether the system escapes every prior condition. It is whether the interaction between its rules, data, and search process yields mathematically significant structures not explicitly supplied as targets.

Artificial systems can therefore become genuine sites of disclosure even without human-like consciousness.

12.7 The Limits of Formal Neutrality

Formal methods often seek neutrality by replacing meaningful names with variables and uninterpreted symbols. This is valuable because it reduces bias and permits generality.

Yet formal neutrality is always bounded.

A symbol may be uninterpreted with respect to its conventional meaning while already classified as a constant, relation, or function of fixed arity. A language may avoid naming addition while preserving composition, substitution, equality, and variable reuse.

The formal system is neutral among some interpretations but not among all possible forms of intelligibility.

A binary operation language privileges functional combination. Equational logic privileges identity statements. Finite model search privileges structures that can be exhaustively represented. A complexity metric privileges certain forms of simplicity.

No formal inquiry opens every mathematical possibility simultaneously.

This does not invalidate formalism. It makes formal scope explicit.

A disciplined system should state:

what distinctions it can represent;

what operations it assumes;

what equivalences it recognizes;

what properties remain inexpressible;

what search boundaries limit disclosure.

Neutrality should be treated as a relative reduction of interpretive commitment, not as the elimination of every prior structure.

12.8 From Abiogenic Algebra to Operation-First Mathematical Phenomenology

The term Algebraic Abiogenesis can be retained only if it is understood as relative emergence. A more exact conceptual framework is operation-first mathematical phenomenology.

This framework begins with operations that are available in use but withheld from familiar classification. It studies how laws, constants, derived operations, and theories become explicit through systematic engagement.

Its object is not the birth of algebra from non-algebra. Its object is the formation of mathematical visibility.

The framework asks:

How does an operation first become available?

Which practices make its behavior intelligible?

Which regularities become conspicuous?

Which equivalences stabilize new objects?

Which observational policies shape the resulting theory?

Which historical comparisons transform anonymous behavior into recognized mathematics?

This program can support rigorous mathematical work. It does not require replacing proofs with philosophical description. Rather, it adds an account of the passage by which a candidate structure enters the domain of proof.

The mathematical core remains universal algebra, model theory, clone theory, automated reasoning, finite classification, and exact computation.

The phenomenological layer explains how those formal objects become disclosed within human and artificial practices.

12.9 What It Means for Algebra to Become Visible

Algebra becomes visible when operational behavior is no longer encountered only as a sequence of isolated results.

A pattern becomes visible as invariant.

An invariant becomes visible as law.

A law becomes visible as part of a theory.

A theory becomes visible as a structure preserved across models and representations.

This visibility is not merely visual. It is conceptual availability. The structure can now be named, manipulated, proved, generalized, and communicated.

The operation table was already determinate. The derived functions were already fixed. The identities were already true or false. But their organization was not yet available as a coherent mathematical object.

The event of disclosure is the formation of that coherence.

Algebra becomes visible when differences of notation, labeling, and presentation are separated from differences of structure. It becomes visible when a short law replaces thousands of cases. It becomes visible when a failed identity reveals the boundary of a theory. It becomes visible when a derived operation reorganizes an opaque table into a familiar system.

The final philosophical conclusion is therefore restrained but substantial.

Mathematics does not arise from absolute nothingness in operation-first experiments. It arises into explicitness.

The world, machine, or table supplies structured possibilities. The investigative practice selects, tests, compresses, and stabilizes them. Proof determines which articulations are valid. Historical comparison determines which are familiar and which may be new.

Algebraic formation is thus neither arbitrary invention nor passive reception. It is the disciplined disclosure of invariant structure through practices capable of making that structure explicit.

What is born is not algebra itself.

What is born is algebra’s visibility.

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